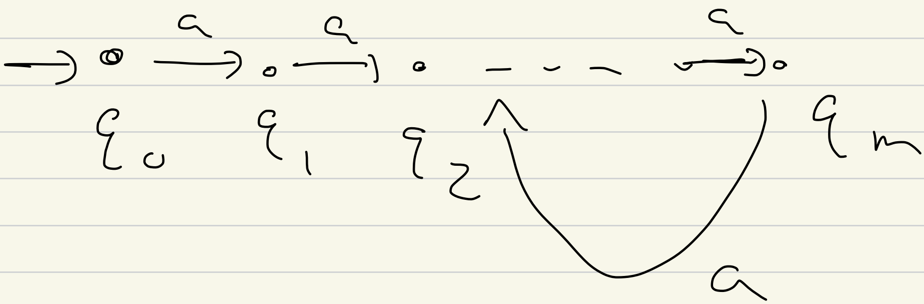
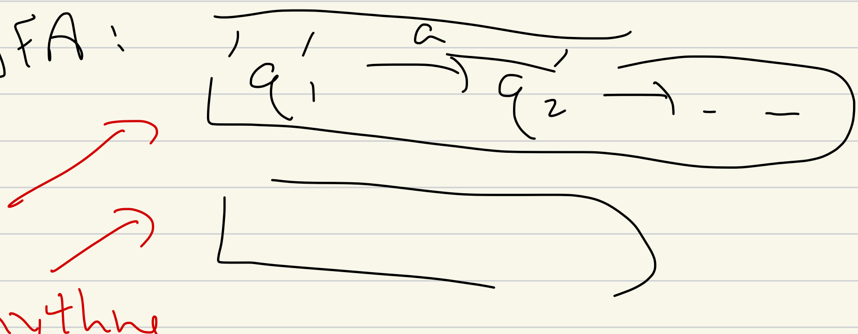
CPSC 421/501 Oct 6, 2023

Homework 5:

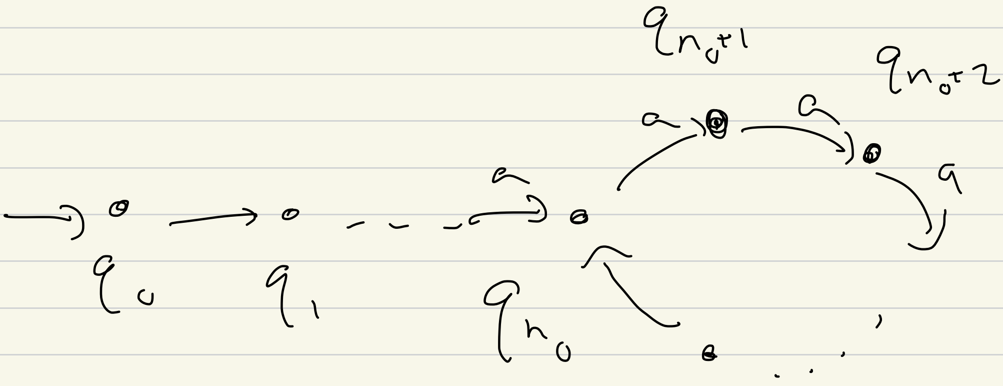
DFA $\Sigma = \{a\}$



DFA:



anything else is irrelevant to the DFA



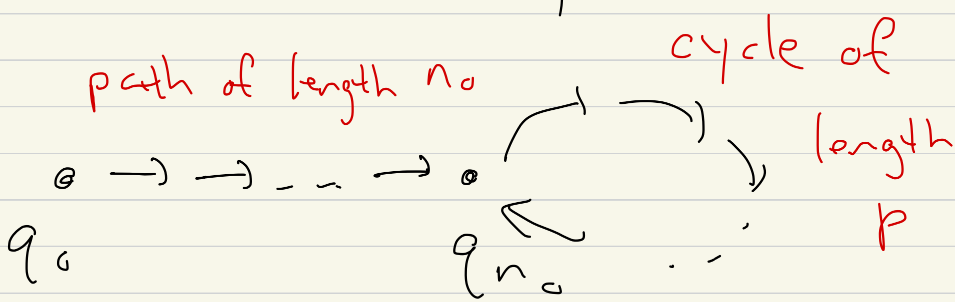
$q_m = q_{n_0+p-1}$

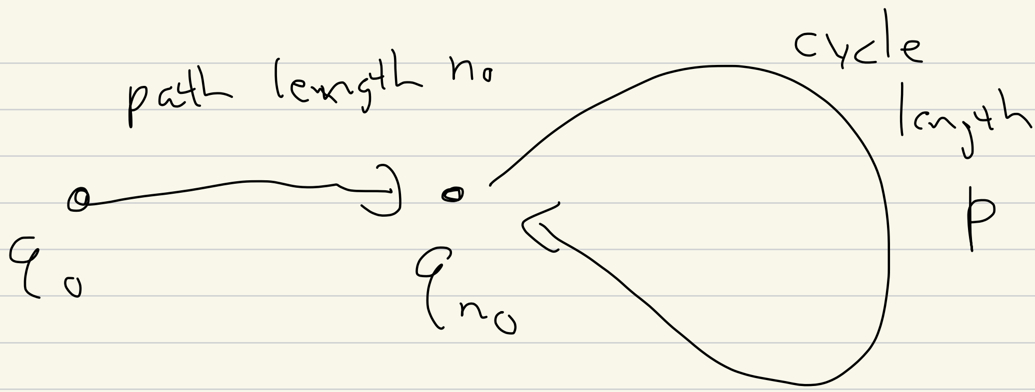
Set p given by

$$m = n_0 + p - 1$$

so $\quad p = m - n_0 + 1$

Graph theoretically





For $n < n_0$

$a^n \rightarrow$ is taken to the state q_n

$\varepsilon \rightarrow q_0, \; a \rightarrow q_1, \; a^2 \rightarrow q_2, \; \ldots$

$a^{n_0} \rightarrow q_{n_0}$

$a^{n_0+p} \rightarrow q_{n_0} \quad \ldots$

Ch 0:

A directed graph is a tuple

$$(V, E, t, h)$$

V ↑ vertices, E ↑ edges

$$t : E \to V$$

$$h : E \to V$$

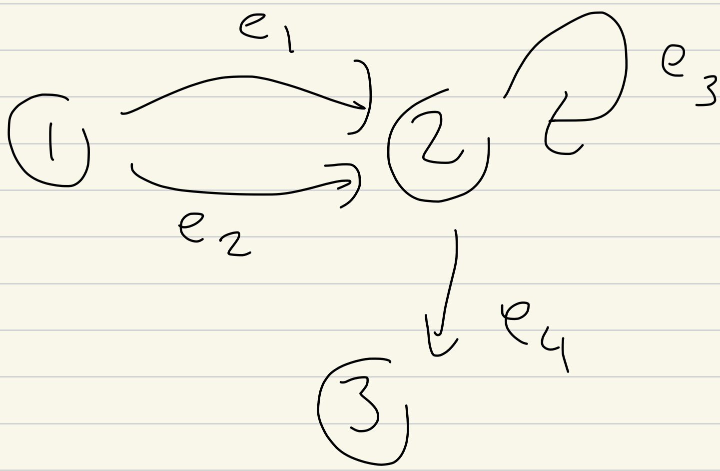
V the "vertex set"

E " "edge set"

$t(e)$ " tail of e

$h(e)$ " head of e

Example



$V = \{ ①, ②, ③ \}$

$E = \{ e_1, e_2, e_3, e_4 \}$

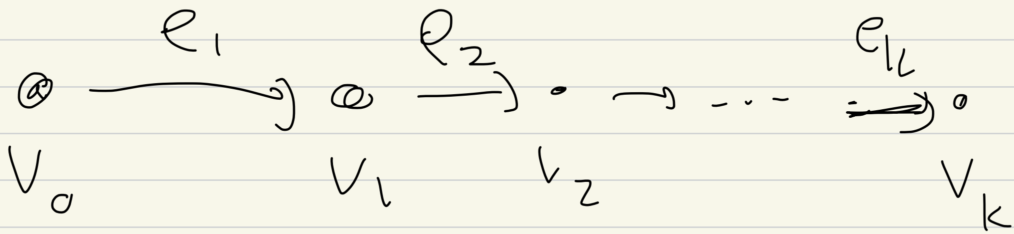
e_1 $\quad t(e_1) = ①, \; h(e_1) = ②$

e_2

e_3

e_4

A *path* in a directed graph



s.t.

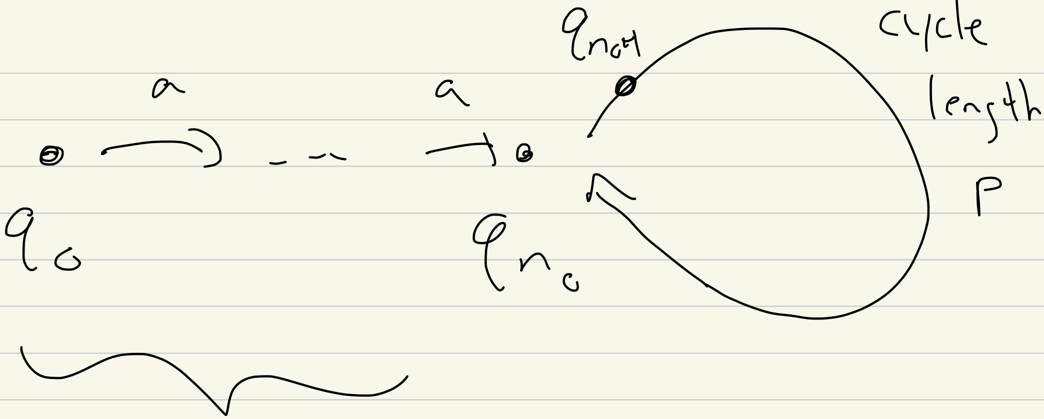
$$h(e_i) = v_i$$

$$t(e_i) = v_{i-1}$$

for all $i = 1, \dots, k$

and s.t. $v_0, \dots, v_k$ are distinct

Cycle: The same, except $v_0 = v_k$

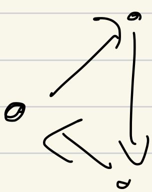


path length n_0

 # vert = $n_0 + p$

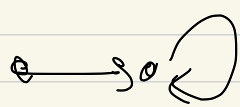
⇕

states in the DFA



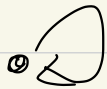
$n_0 = 0$, $p = 3$

#V = 3



$n_0 = 1$, $p = 1$

#V = 2



$p = 1$ $\qquad V = 1$

$n_0 = 0$

$a_{n_0} \to q_{n_0}, \quad a_{n_0+p} \to q_{n_0}$

$a^{n_0}, a^{n_0+p}, a^{n_0+2p}, \ldots \to q_{n_0}$

$a^{n_0+1}, a^{n_0+1+p}, a^{n_0+1+2p}, \ldots \to q_{n_0+1}$

So any of $q_0, q_1, \ldots q_{n_0+1},$

$\ldots q_{n_0+(p-1)},$

could be $\left\{ \begin{array}{l} \text{accepting} \\ \text{rejecting} \end{array} \right\}$

If $n \geq n_0$, then

$$a^n \in L \iff a^{n+p} \in L$$

i.e.

$a^{n_0}, a^{n_0+p}, a^{n_0+2p}, \ldots$ either $\begin{cases} \text{all in } L \\ \text{all outside } L \end{cases}$

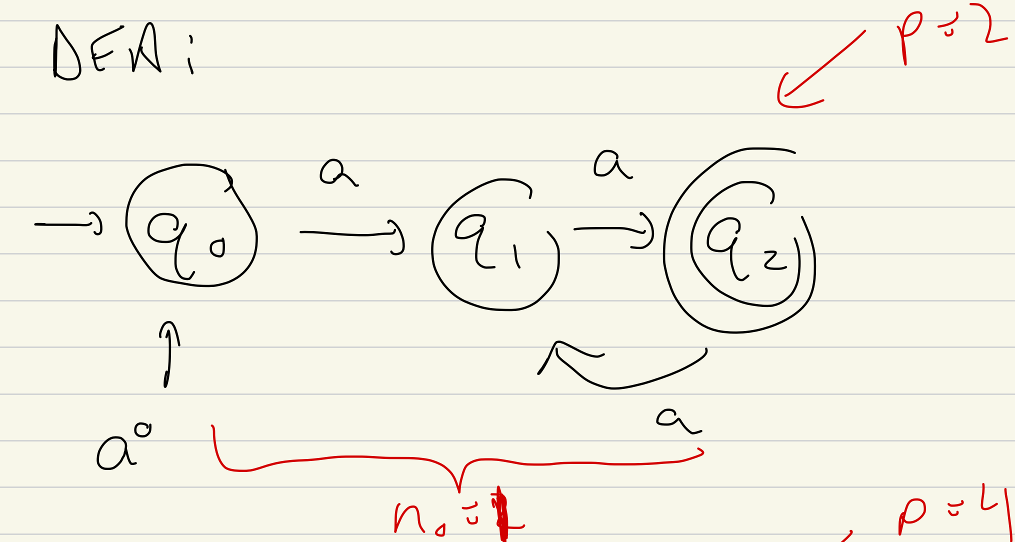
$a^{n_0+1}, a^{n_0+p+1}, \ldots$ " " " "
" " " "

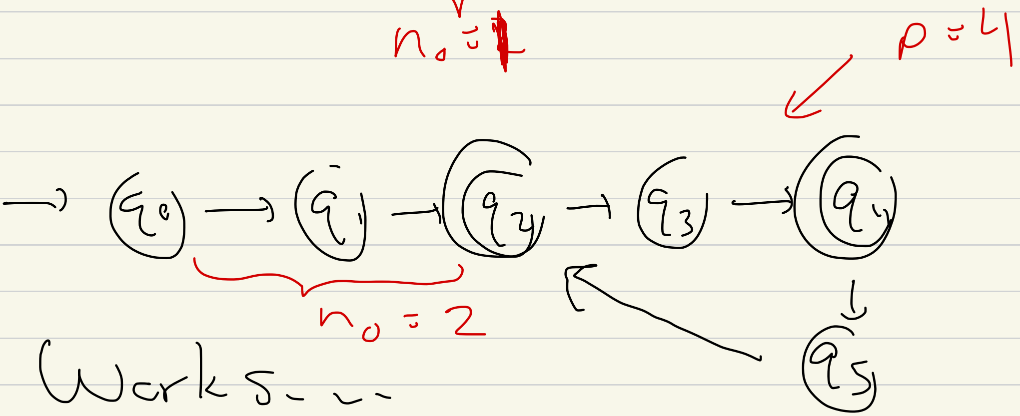
$\vdots$

$\{ a^{2k} \mid k \in \mathbb{N} \}$

$= \{ a^2, a^4, a^6, \dots \}$

DFA:

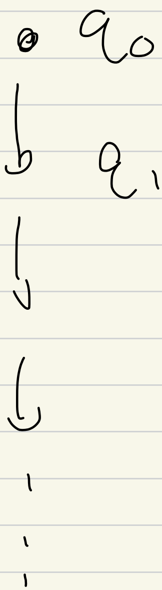


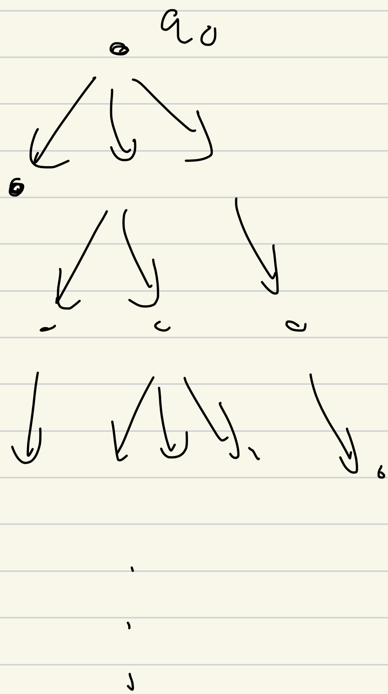


Works...

not minimal # states

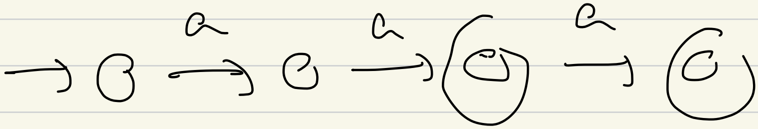
Review NFA





DFA

$\{a^2, a^3\}^*$

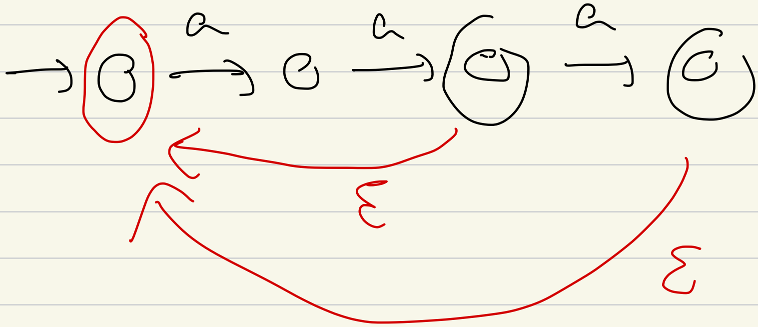


↑ nowhere to go

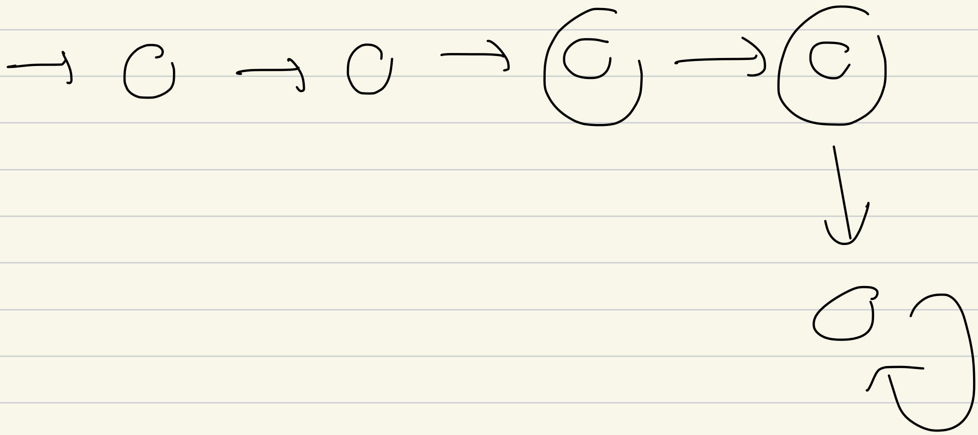
$\{a^2, a^3\}$

↓

$\{a^2, a^3\}^*$



DFA $\{a^2, a^3\}$



$p = 1$