CPSC 421/501 Oct 4, 2023

Last time: languages like

$\{a^3, a^5\}^*$, $\{a^5, a^7\}^*$, $\{a^6, a^{10}\}^*$

are a bit mysterious.

Today: If L_1, L_2 are regular, then

$L_1 \cup L_2$, $L_1 \circ L_2$, L_1^*

are also regular.

Leads to: NFA (N = non-deterministic)

and regular expressions

1.3 [Sip]

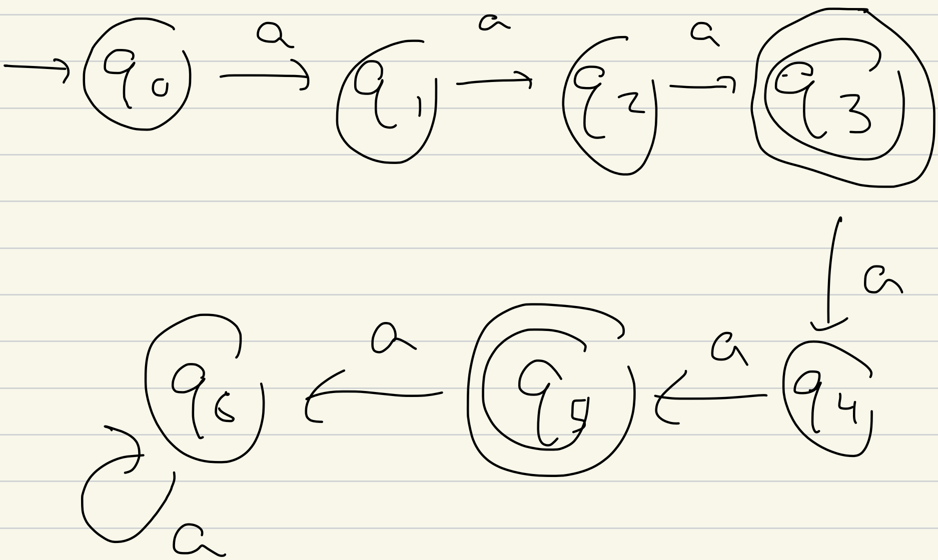
1.2 [Sip]

Goal: write a <u>DFA</u> for

$$L^+ = \varepsilon \cup L^1 \cup L^2 \cup L^3 \cup \ldots$$

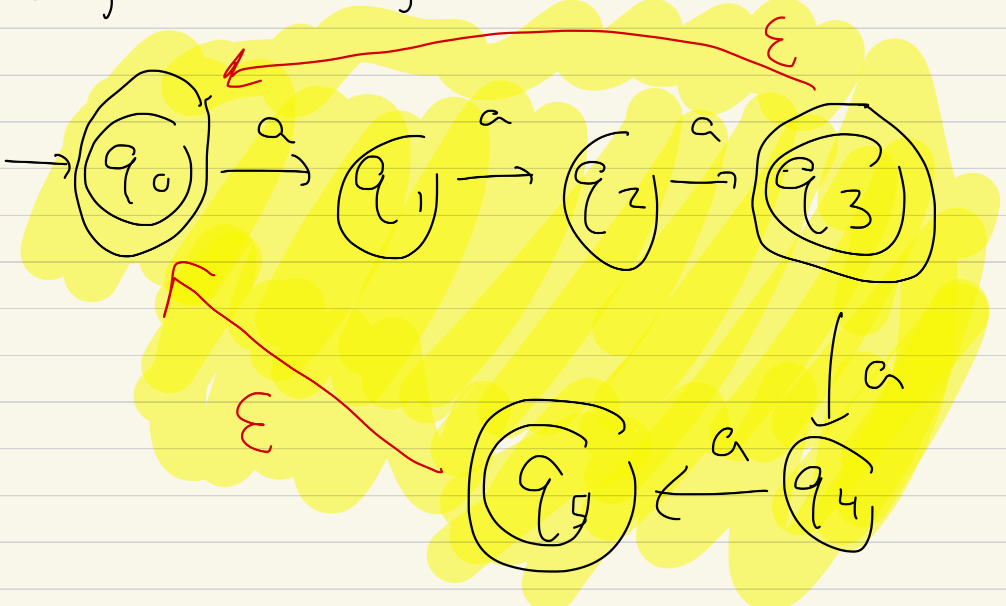
where

$\boxed{L = \{a^3, a^5\}}$, recognized by

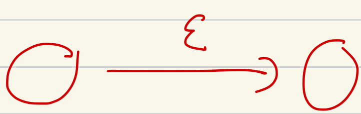


Trick / method:

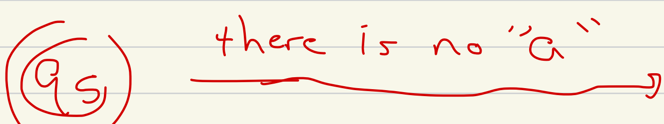
slight strengthen our DFA's:

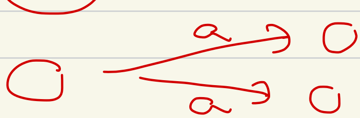


Allow:

1. 

"jump" or "empty string transition"

2. 

3. 

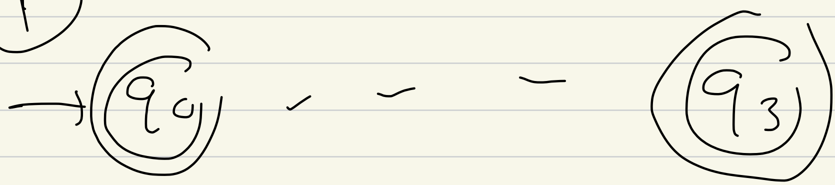
allow same symbol to lead to many states

$L^* = \{a^3, a^5\}^*$

say we want to accept $a^6 \in L^2$

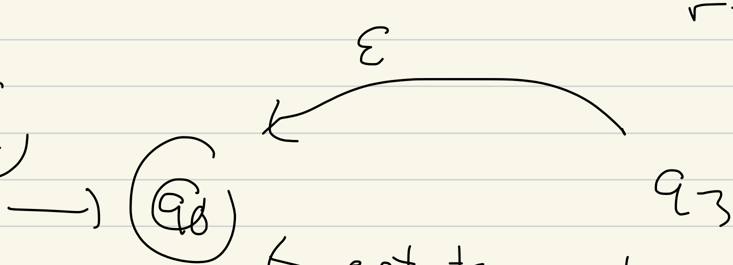
$a^6 = a^3 \circ a^3, \quad a^3 \in L;$

1



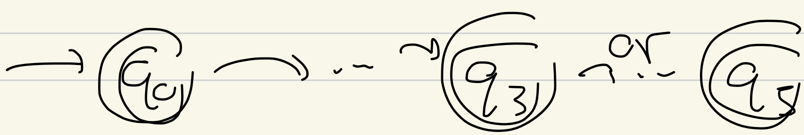
get to here reading a^3

2



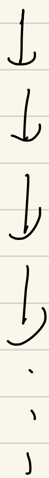
get to q_0 having read a^3

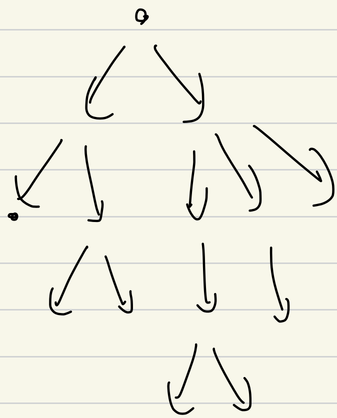
3



accept $a^3 \circ a^3$ accept $a^3 \circ a^5$

In $\{s_{i_p}\}$:





deterministic non-deterministic

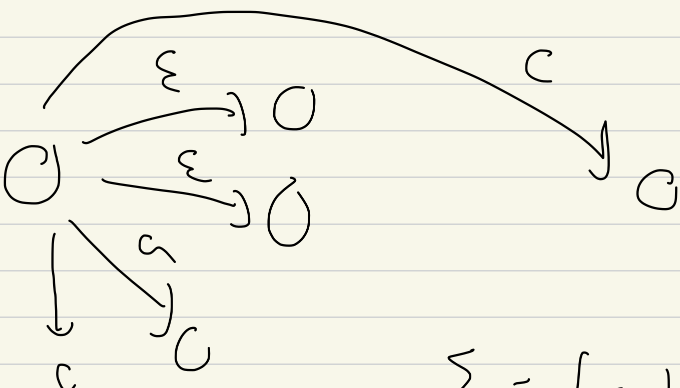
=

Define the language recognized by an <u>NFA</u> to be $w \in \Sigma^*$ s.t. there is at least one accepting path

Formally, an NFA is a tuple $(Q, \Sigma, \delta, q_0, F)$

where

$$\delta : Q \times (\Sigma \cup \{\varepsilon\}) \to \text{Power}(Q)$$



$\Sigma = \{a, b, c\}$

Each NFA, $M = (Q, \Sigma, \delta, q_0, F)$ recognizes

$$L = \{ w \in \Sigma^* \mid \text{there is at least one allowable sequence of transitions in } M \text{ that takes } w \text{ to a state in } F \}$$

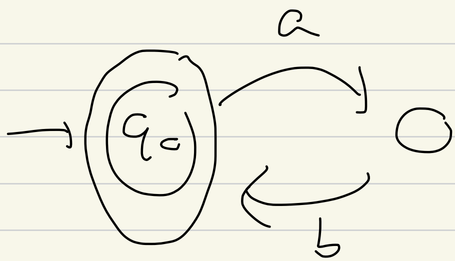
For DFA's

$$L = \{ w \in \Sigma^* \mid w \text{ is taken to a state in } F \text{ by } M \}$$

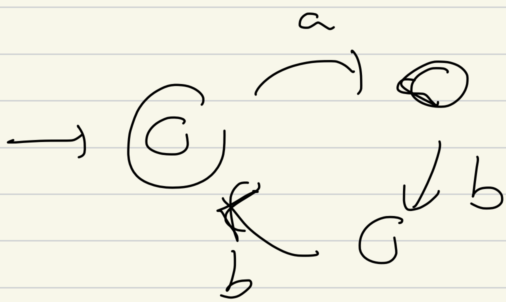
So say

$L_1 = \{ab\}^*$

NFA:

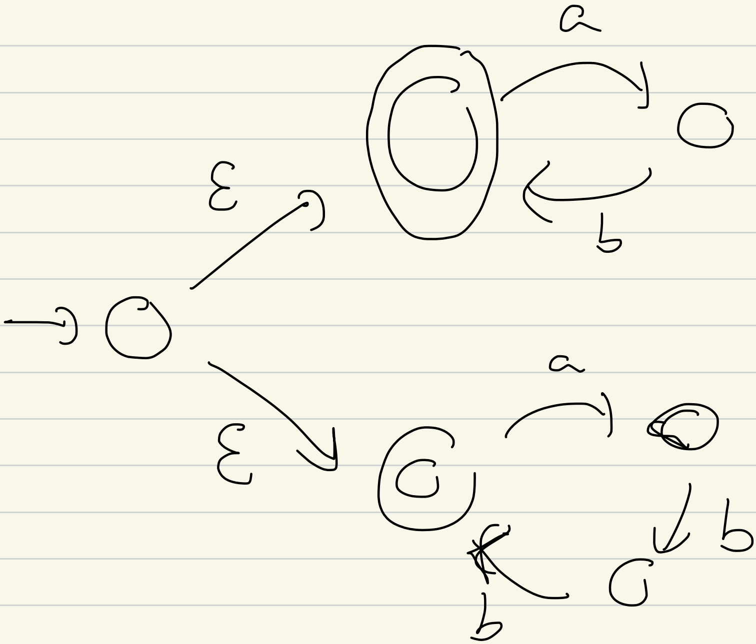


$L_2 = \{abb\}^*$

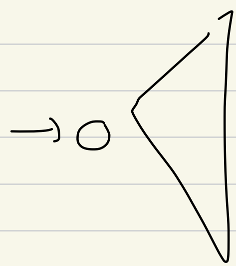


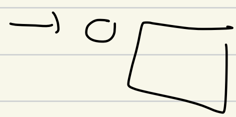
$L_1 \cup L_2$

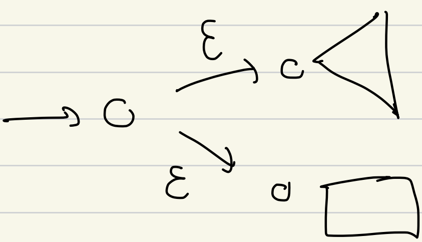
$L_1 \cup L_2$



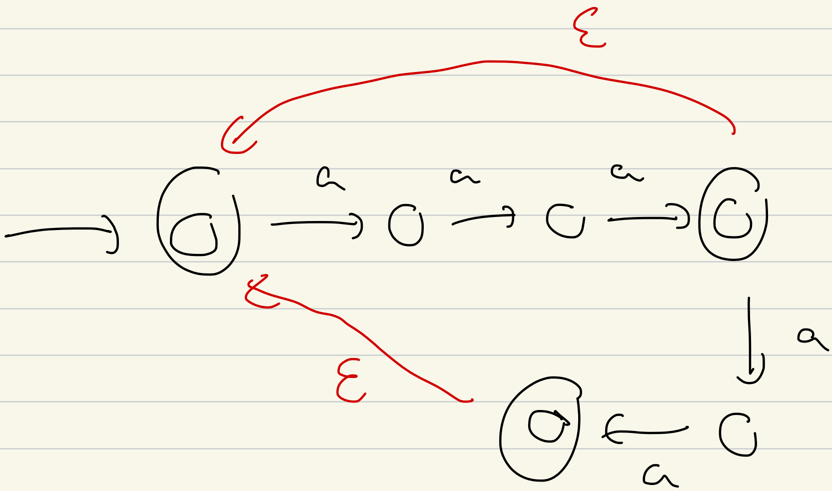
Similarly

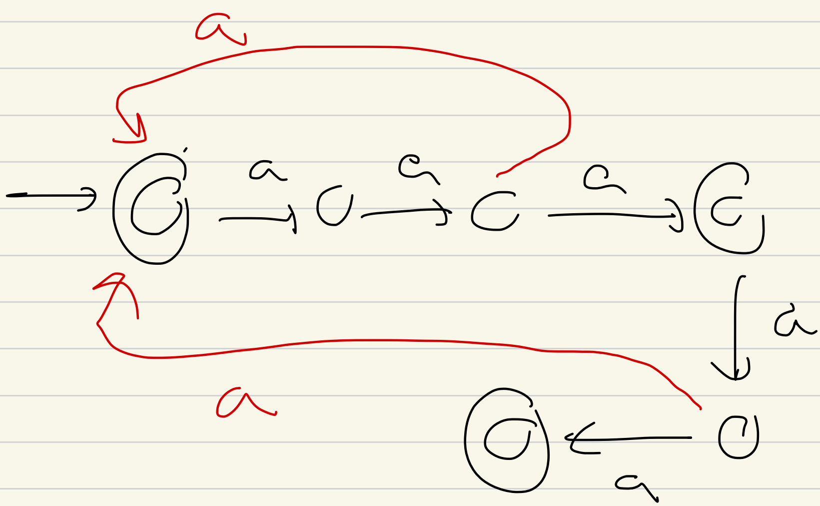
L_1 recognized by 

L_2 " " 



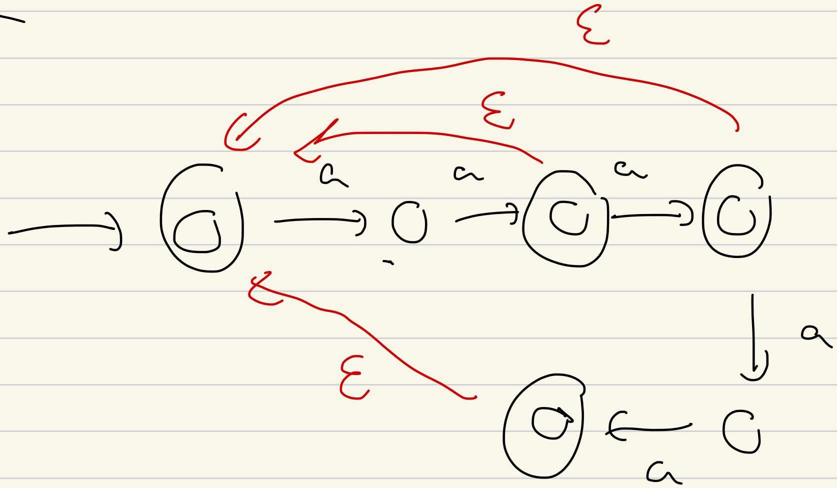
$L = \{a^3, a^5\}$





Say $L = \{a^2, a^3, a^5\}$

L^*



<u>Thm:</u> If L is recognized by an NFA, then L is regular, i.e. there is a DFA recognizing L.

Pf: Given an NFA,

$M = (Q, \Sigma, \delta, q_0, F)$

let

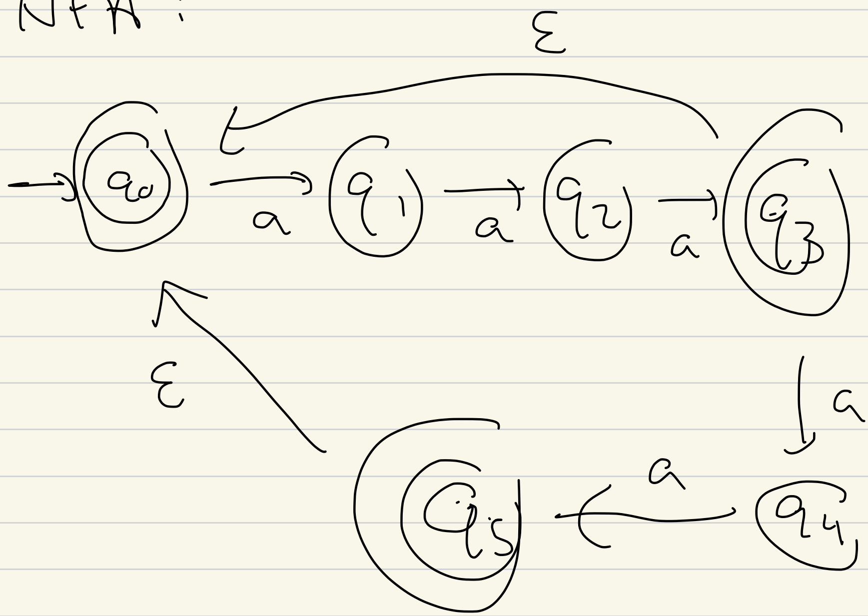
$$M' = \left(\text{Power}(Q), \Sigma, \tilde{\delta}, \tilde{q}_0, \tilde{F} \right)$$

where

(1) an element of Power(Q) means "all possible elements of Q where you could be after reading the first so many inputs"

e.g.

NFA:



Read: ε: we can be in $\{q_0\}$

a — — — — $\{q_1\}$

a^2: " " " $\{q_2\}$

a^3: " " " $\{q_3, q_0\}$

a^4: " " " $\{q_4, q_1\}$

read a^5: we can be in $\{q_5, q_0, q_2\}$

a^6: " " " " $\{q_1, q_3, q_d\}$

We accept the "states"

of Power(Q), i.e.

$Q' \subset Q$ iff $|Q' \cap F| \geq 1$.