Last time: languages like
\{ a^3, a^5 \}^*, \{ a^5, a^7 \}^*, \{ a^6, a^{10} \}^*
are a bit mysterious.

Today: If \( L_1, L_2 \) are regular,
then
\( L_1 \cup L_2, L_1 \cap L_2, L_1^* \)
are also regular.

Leads to: NFA (\( N = \) non-deterministic)
and regular expressions
\( L_1 \cdot L_2, L_1 L_2 \).
Goal: write a DFA for
\[ L^+ = \varepsilon \cup L \cup L^2 \cup L^3 \ldots \]

where
\[ L = \{ a^3, a^5 \} \]
recognized by

\[ \begin{align*}
&\rightarrow q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \\
&\rightarrow q_6 \xrightarrow{a} q_5 \xrightarrow{a} q_4
\end{align*} \]
Trick/method:

Slightly strengthen our DFA's:

Allow:

1. $\varepsilon \rightarrow \varepsilon$

   "jump" or "empty string transition"

2. $\varepsilon \rightarrow$?

3. $a \rightarrow a$

   allow same symbol to lead to many states
\[ L^k = \{a^3, a^5\} \]

Say we want to accept \( a^6 \in L^2 \)

\[ a^6 = a^3 \circ a^3, \quad a^3 \in L \]

1. \( \rightarrow \) 90
2. \( \rightarrow \) 90
3. \( \rightarrow \) \( \rightarrow \) 93
4. \( \rightarrow \) \( \rightarrow \) 93

Get to here reading \( a^3 \)

\( \rightarrow \) \( \rightarrow \) 90 leaving read \( a^3 \)

\( \rightarrow \) \( \rightarrow \) 93 or \( \rightarrow \)
Define the language recognized by an NFA to be $w \in \Sigma^*$ s.t. there is at least one accepting path.
Formally, an NFA is a tuple \((Q, \Sigma, \delta, q_0, F)\) where

\[
\delta : Q \times (\Sigma \cup \{\varepsilon\}) \rightarrow \text{Power}(Q)
\]

\[\Sigma = \{a, b, c\}\]
Each NFA, \( M = (Q, \Sigma, \delta, q_0, F) \) recognizes

\[
L = \{ w \in \Sigma^* \mid \text{there is at least one allowable sequence of transitions in } M \text{ that takes } w \text{ to a state in } F \}
\]

For DFA's

\[
L = \{ w \in \Sigma^* \mid w \text{ is taken to a state in } F \text{ by } M \}
\]
So say

$L_1 = \{ ab \}^*$

**NFA:**

$L_2 = \{ abb \}^*$

$L_1 \cup L_2$
Similarly, \( L_1 \) recognized by \( A \) and \( L_2 \).
$L = \{ a^3, a^5 \}$
Say $L = \{ a^2, a^3, a^5 \}$

If $L$ is recognized by an NFA, then $L$ is regular, i.e. there is a DFA recognizing $L$. 

Thm!
Pf: Given an NFA,
\[ m = (Q, \Sigma, \delta, q_0, F) \]

let
\[ m' = (\text{Power}(Q), \Sigma, \tilde{\delta}, \tilde{q}_0, F) \]

where

1) an element of \( \text{Power}(Q) \)

means "all possible elements of \( Q \) where you could be after reading the first so many inputs"
e.g.,

NFA:

\[ q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \]

\[ q_3 \xrightarrow{a} q_4 \]

Read! \( \varepsilon \) we can be in \( \{ q_0 \} \)

\[ a \xrightarrow{a} \{ q_1 \} \]

\[ a^2 \xrightarrow{a} \{ q_2 \} \]

\[ a^3 \xrightarrow{a} \{ q_3, q_0 \} \]

\[ a^4 \xrightarrow{a} \{ q_4, q_1 \} \]
read $a^5$, we can be in \( \{q_5, q_0, q_2\} \)

\[ a^6 ! " " " " \{ q_1, q_3, q_4 \} \]

We accept the "states"
of \( \text{Power}(Q) \), i.e.

\[ Q' \preceq Q \text{ iff } |Q' \cap F| \geq 1 \]