CPSC 421/501 Sept. 29, 2023

Today:

- Regular and non-regular languages over $\Sigma = \{a\}$
- Regular Expressions and Regular Languages, §1.2 NFA's

motivation $\longrightarrow$ §1.3 Regular Expressions

Specifically:

Define $\cup$, $\circ$, $*$, i.e.

$L_1 \cup L_2$, $L_1 \circ L_2$, L_1^*

prove each is regular if L_1, L_2 are regular.

Example: $(\{a^5\} \cup \{a^7\})^* = ???$

Note: In [Sip], §1.3, a regular expression is

- $\emptyset$, ε, alphabet symbol
- A
  - union ($\cup$)
  - concatenation ($\circ$)
  - star ($*$)

  of other regular expressions

<u>We don't allow $\neg$ (negation)</u>

Examples: if $\Sigma = \{a, b, c, u\}$,

$$\Sigma^* ubc\, \Sigma^* = \Sigma^* \circ u \circ b \circ c \circ \Sigma^*$$

$$\Sigma^* (ubc \cup cba)\, \Sigma^*$$

On the homework:

give a DFA:

$$(Q, \Sigma, \delta, q_0, F)$$

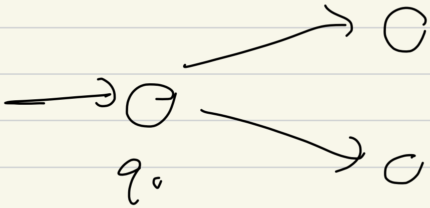
sets

δ:

- Give the values of $\delta(q, \sigma)$
- " a table

$\delta(q,\sigma)$	$\sigma = a$	$\sigma = b$
$q = q_0$	q_{--}	
$q = \vdots$	q_{--}	
$\vdots$		

- or



$\Sigma = \{a\}$.

<u>Define</u>: If L_1, L_2 are languages over Σ, then

$L_1 \cup L_2$ = usual union as sets

$$L_1 \circ L_2 = \left\{ w_1 \circ w_2 \;\middle|\; \begin{array}{l} w_1 \in L_1 \\ w_2 \in L_2 \end{array} \right\}$$

$$L_1^* = \varepsilon \cup L_1 \cup (L_1 \circ L_1) \cup (L_1 \circ L_1 \circ L_1) \cup \cdots$$

require NFA, §1.2

Example: $\Sigma = \{a\}$,

$L_1 = \{aaaaa\} = \{a^5\}$

$L_2 = \{a^7\}$

$(L_1 \cup L_2)^* = \{a^5, a^7\}^*$

$= \{\varepsilon, a^5, a^7, a^5a^5, \underbrace{a^5}\underbrace{a^7}, \underbrace{a^7}\underbrace{a^5},$

$a^7a^7, a^5a^5a^5, a^5a^5a^7, \ldots\}$

$=$

$\{aa, bba\}^* = \{\varepsilon, aa, bba, aaaa,$

$\underbrace{aa}\underbrace{bba}, \underbrace{bba}\underbrace{aa}, bbabba, \ldots\}$

$$aaaaa = (a,a,a,a,a)$$

$$= (a)\circ(a)\circ(aaa)$$

$$\left.\begin{array}{l} \underbrace{aa}\,\underbrace{bb}a \\ \downarrow \\ (a,a)\circ(b,b,a) \end{array}\right\} \quad aabba$$

$$\{a^5, a^7\}^*$$

$$= \{\varepsilon, a^5, a^7, a^5a^5, a^5a^7, a^7a^7,$$

$$a^5a^5a^5, a^5a^5a^7, \ldots$$

$$= \{\varepsilon = a^0, a^5, a^7, a^{12}, a^{14}, a^{15},$$

$$a^{17}, \ldots \text{ ??}$$

$(a^5)^p(a^7)^q$ gets a^n for all n sufficiently large,

$\exists n_0$ s.t. if $n \geq n_0$, $a^n \in \{a^5, a^7\}^*$

$$\{a^3, a^5\}^* = \{a^0, a^3, a^5, a^{3+3}, a^{3+5}, a^{5+5}, a^{3+3+3}, a^{3+3+5}$$

$$= \{a^0, a^3, a^5, a^6, a^8, a^9, a^{10}, a^{11}, a^{12}, a^{13}, a^{14}, \ldots\}$$

Not a^7

observe: if $a^n \in \{a^3, a^5\}^*$

then $a^{n+3} \in \{a^3, a^5\}^*$

$$\{b^6, b^{10}\} = \{b^0, b^6, b^{10}, b^{12}, b^{16}, b^{18}, b^{20}, \ldots\}$$

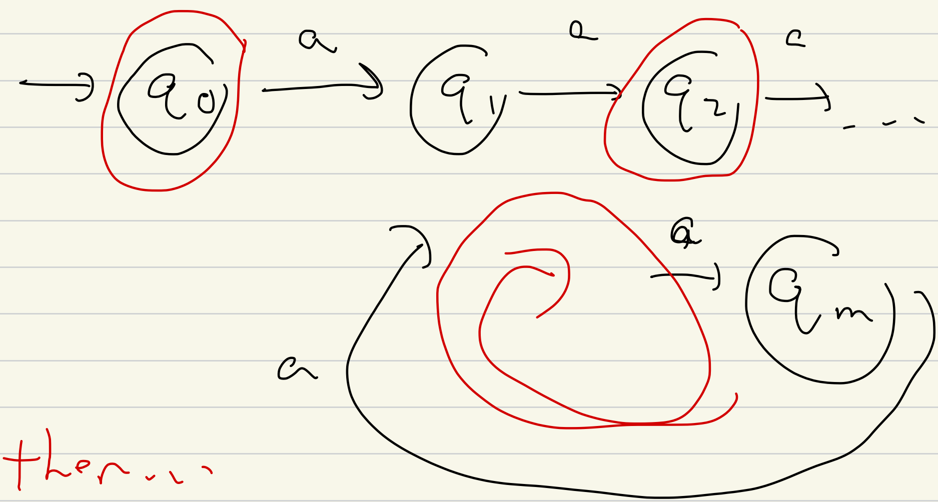
$b \leadsto a^2$

$\{a^{23}, a^{53}, a^{191}\}^* = \ldots$

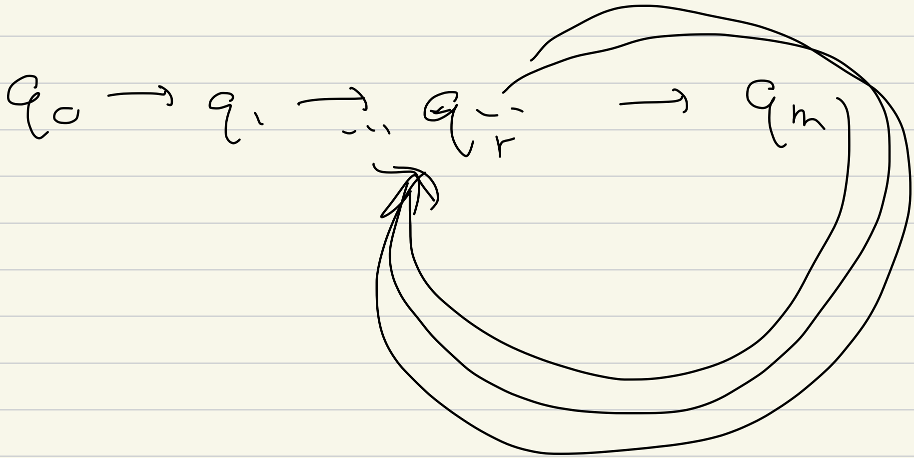
($*$ is a rather serious operation)

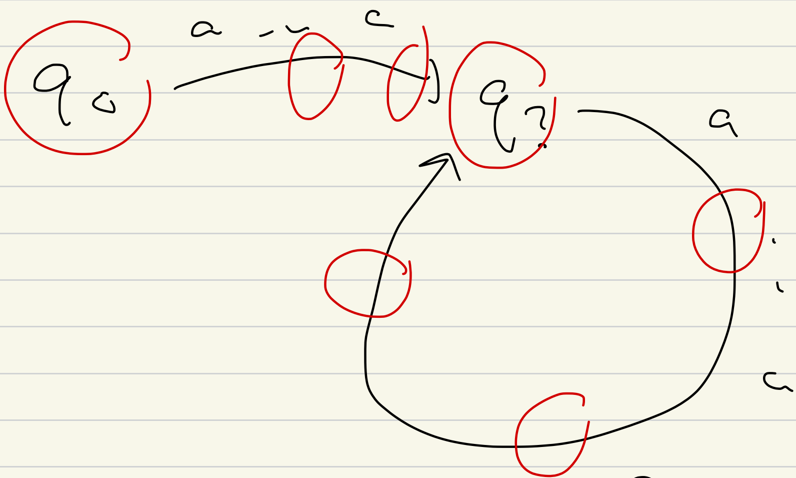
$\Rightarrow$

Rem: $\Sigma = \{a\}$, what is a DFA over Σ?



then...





if length cycle is p

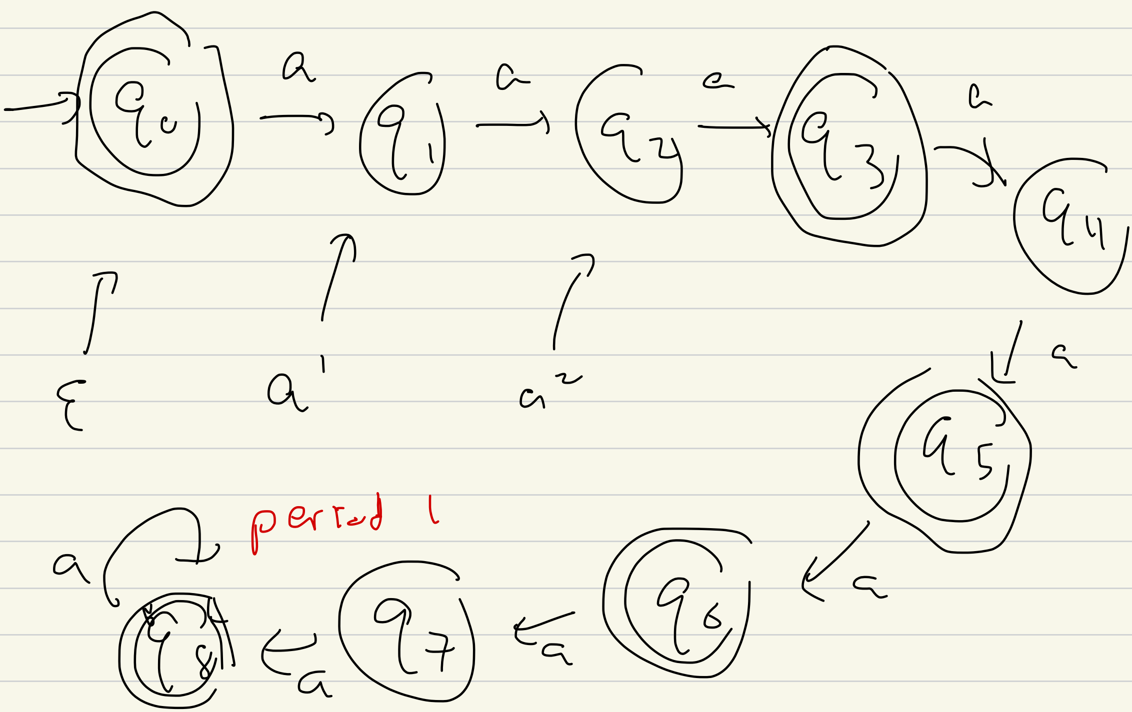
then for sufficiently large n

$$a^n \in L \iff a^{n+p} \in L$$

where L = language recognized by DFA.

?

e.g. $\{a^3, a^5\}^* = \{\varepsilon, a^3, a^5, a^6, a^8, a^9, a^{10}, \ldots\}$



$4-1=3$ $9-4=5$ $16-9=7$

So, is

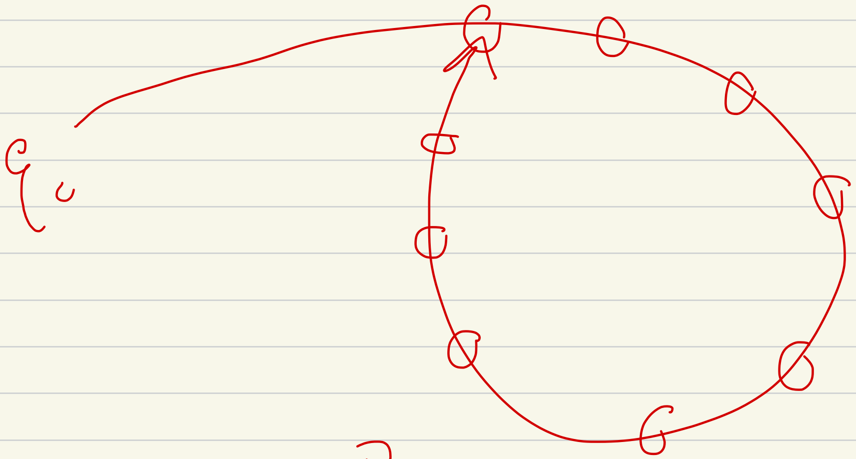
$$L = \{ a^1, a^4, a^9, a^{16}, a^{25}, \ldots \}$$

$$= \{ a^n \mid n \text{ is a } \cancel{\text{perfect}} \text{ square} \}$$

$$= \{ a^{(k^2)} \mid k \in \mathbb{N} \}$$

regular?

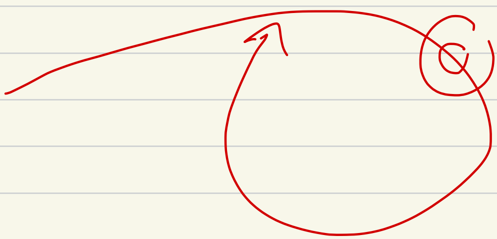
No: L is infinite, so DFA ~~recognizing~~ L looks like



Since L is infinite,

one of the states along the cycle has to accept

say



but then $a^{n_0}, a^{n_0+p}, a^{n_0+2p}, \ldots \in L$

Next time: if L is regular,

then so is L^*.

Via

Non-deterministic finite automata

N F A

National Truth and Reconciliation Day

Saturday, Sept 30

Observed Monday, Oct 2

- It is not easy for survivors of the Indian Residential School System to talk about their past trauma.
- Survivors and their families tire from giving repeated explanations
- Children are not responsible for the mistakes of their parents, but have the obligation to learn about these mistakes
- One of my favourite suggestions "Learn for yourself"