CPSC 421/501 Sept 20, 2023

Last time: $L \subset \Sigma^*_{ASCII}$ is:

<u>decidable</u> if for some p: for all $i \in \Sigma^*_{ASCII}$

$$p \text{ on input } i = \begin{cases} \text{"yes"} & \text{if } i \in L \\ \text{"no"} & \text{if } i \notin L \end{cases}$$

<u>recognizable</u> if same, but

$$p \text{ on input } i \begin{cases} = \text{"yes"} & \text{if } i \in L \\ \neq \text{"yes"} & \text{if } i \notin L \end{cases}$$

Today: "Runnig algorithms in parallel".

AND Paradoxes

Concretely: L^c or L^{comp} or $\overline{L}$

Thm 1: Let L and $\overbrace{\Sigma^*_{ASCII} \setminus L}$ be recognizable, then L is decidable.

Thm 2: ACCEPT_SOME_INPUT is recognizable.

ACCEPT_SOME_INPUT

$$= \{ p \in \Sigma^*_{ASCII} \mid p \text{ accepts some (at least one) input} \}$$

Thm 1 Proof!

Example:

$$\text{"INT} \leq 5\text{"} = \{ i \in \Sigma_{ASCII}^* \mid \underbrace{i \text{ represents an integer} \leq 5}_{\text{short description}} \}$$

verySilly.py = p

$$p \text{ on input } i = \begin{cases} \text{"yes" if } i \in \text{INT}\leq 5 \\ \text{"no" or "loops" if } i \notin \text{INT}\leq 5 \end{cases}$$

say we have another q (equally silly, s.t.)

$$q \text{ on input } i = \begin{cases} \text{"yes"} & \text{if } i \notin INT_{\leq 5} \\ \text{"no", "loops"} & \text{if } i \in INT_{\leq 5} \end{cases}$$

Given i

run p for 1 step ~~on~~ i
" q " 1 step on i
" p for 2 steps on i
" q " " " " i
⋮
p " 30 " " "
q " 30 " " "

If $L \subset \Sigma^*$, we write

L^{comp} (L^c or $\overline{L}$) for the

<u>complement</u> of L

$$\Sigma^* \setminus L$$

here Σ has to be understood if we write L^{comp}!

Example

$$PALINDROME_{\{a,b\}} = \{\varepsilon, a, b, aa, bb, \ldots\}$$

subset of $\{a,b\}^* \subset \{a,b,c\}^*$

Claim:

ACCEPT_SAME_INPUT is recognizable.

$=$

$$\text{ACCEPTANCE} = \{ p \,\sigma_0\, i \mid p \text{ accepts } i \}$$

Enumerate $\{a,b\}^*$

$i_1 = \varepsilon$

$i_2 = a$

$i_3 = b$

$i_4 = aa$

$i_5 = ab$

$i_6 = ba$

$i_7 = bb$

$i_8 = aaa$

$\vdots$

So $n \longmapsto i_n \in \{a,b\}^*$

really $\mathbb{N} \xrightarrow[\text{bijection}]{} \{a,b\}^*$

Phase 1: run p on i_1 1 step

" 2: " " " i_1 2 steps

and i_2 1 step

Phase 3 run p on i_1 3 steps

on i_2 2 steps

on i_3 1 step

⋮

Rem 1: Our version of Python alg

comes from Turing Machines

(Ch 3 of [Sip])

So allow only 1 thread

of a sequential computation

Phase 1	takes	1	$\binom{2}{2}$	(Quadratic)
Phase 2	"	3	$\binom{3}{2}$	
Phase 3	"	6	$\binom{4}{2}$	
Phase 4	"	10	$\binom{5}{2}$	

Say i_{37} on p halts after 100

total cubies in $(37+100)$

speed up ?

Paradoxes or Contradictions !

Cantor's Thm:

self-reference

$$T = \{ s \mid s \notin f(s) \}$$

negation

Assume $f(t) = T$ for some t ...

"I am lying."

negation

self-reference

Berry Paradox (likely of B. Russell):

"Let n be the smallest positive integer not described by an English phrase of fewer than $\underbrace{500}_{\text{fifty}}$ words"

Paradox: What is n ?

Say

$$n = 10^{100} - 37 + 101^2$$

Russell's Paradox:

Let

$$T = \boxed{\left\{ \text{all sets } S \text{, such that } S \notin S \right\}}$$

e.g.

$\{1, 2\}$ is not an element of $\{1, 2\}$

since

$$\{1,2\} \neq 1$$
$$\{1,2\} \neq 2$$

Does $T \in T$?

If $T \in T \;=\; \{\, S \mid S \notin S \,\}$

then $T \notin T$ contradiction

If $T \notin T$ so $T \in T$