CPSC 421/501 Sept 18, 2023

Today: more examples of "unsolvable" "problems" using reductions.

"Problem" means a language

(i.e. a "decision problem")

(i.e. an $L \subset \Sigma^*$, some Σ)

"Unsolvable" means either

(1) undecidable

OR

(2) unrecognizable

Last time:

GROUCHO_MARX_SELF

=

NON_SELF_ACCEPTING

=

$$\{ p \in \Sigma^*_{ASCII} \mid p \notin \text{LanguageRecBy}(p) \}$$

$$\text{LangRecBy}: \Sigma^*_{ASCII} \to \text{Power}(\Sigma^*_{ASCII})$$

$$\text{LangRecBy}(p) = \begin{cases} \{ i \mid p \text{ accepts } i \} \\ \emptyset \end{cases}$$

if p is valid Python prog

Cantor's Thm $\Rightarrow$

$\{ p \mid p \notin \text{LangRecBy}(p) \} = T$

is unrecognizable.

We used a "reduction" to show

that

NON-ACCEPTANCE

$= \{ p\,\sigma_0\,i \mid p \text{ does not accept } i \}$

Our convention of describing a string p, followed by another string i, $\sigma_0 = \langle \text{BELL} \rangle$

$\overline{\text{pair}}$ is really some way to express $\langle p, i \rangle$, i.e.

a description of a pair (p, i)

where we anticipate:

p = Python program $\in \Sigma^*_{ASCII}$

i = some input to p $\in \Sigma^*_{ASCII}$

We proved that

if NON-ACCEPTANCE were recognizable then T = GROUCHO_MARX_SELF OR NON-SELF-ACCEPTANCE would also be recognizable.

Decidable vs. recognizable

- We say Python program, p, is a <u>decider</u> if on any $i \in \Sigma^*_{ASCII}$, p on input i is either

or "yes" — p accepts i
"no" — p rejects i

(other possibilities are called "loops", i.e. p loops on i)

=

$L \subseteq \Sigma^*_{ASCII}$ is decidable if some decider, p, recognizes L, i.e.

$$L = \text{LanguageRecBy}(p)$$

$$= \{ i \mid p \text{ accepts } i \}$$

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e.g.

$$\text{ACCEPTANCE} = \{ p \sigma_0 i \mid p \text{ accepts } i \}$$

and

SELF-ACCEPTANCE

$$= \{ p \mid p \in \text{LangRecBy}(p) \}$$

are recognizable (!!)

What is

$$\text{LangRecBy}(\text{verySilly.py})$$

$$= \left\{ \langle n \rangle \;\middle|\; \begin{array}{c} n \in \mathbb{Z} \text{ s.t.} \\ n \leq 5 \end{array} \right\}$$

$$= \left\{ \begin{array}{c} \text{strings that represent} \\ \text{integers} \leq 5 \end{array} \right\}$$

verySilly.py is <u>not</u> a decider.

But:

ACCEPTANCE = $\{ p \sigma_0 i \mid p \text{ accepts } i \}$

is { undecidable / recognizable

Why recognizable?

Just

(1) simulate p on input i

(2) run Python -m pdb p (input i)

run debugger ...

(3) also called <u>universal</u> Python prog

Proposition: If L is decidable, then $\Sigma^* \setminus L$ is decidable

= "complement of L"

$A \setminus B$ = A "set minus" B

$= \{ a \in A \mid a \notin B \}$

Proof: If p decides L

$p = \left\{ \begin{array}{l} \vdots \\ \text{return("no")} \\ \vdots \\ \text{return("no")} \\ \vdots \\ \text{return("yes")} \\ \vdots \end{array} \right\}$. $\left\{ \begin{array}{l} \vdots \\ \text{return("yes")} \\ \text{-- ("yes")} \\ \text{("no")} \end{array} \right\}$ @ q

Hence p says "yes" iff q says "no"

" " "no" " " "yes"

hence

$$\text{LangRecBy}(q) = \Sigma^* \setminus \text{LangRecBy}(p)$$

<u>Prop</u>: If L is decidable (by p)

then L " recognizable. (by p)

$\Longrightarrow$

PROG_THAT_ACCEPT_AT_LEAST_ONE_INPUT

$$= \{ p \mid p \text{ accepts some } i \in \Sigma^*_{ASCII} \}$$