CPSC 421/501 September 15, 2023

We defined

$$\text{LanguageRecBy} : \Sigma^*_{ASCII} \to \text{Power}(\Sigma^*_{ASCII})$$

GROUCHO-MARX-SELF =

$$T = \{ p \in \Sigma^*_{ASCII} \mid p \notin \text{LanguageRecBy}(p) \}$$

is not recognizable, i.e. not in the image of LanguageRecBy(p).

Today: - Why? (Proof?)

- So what?

Answer to "So What?" involves "reductions"

VALID-PYTHON-PROGRAMS

$\subset \Sigma^*_{ASCII}$

We will talk about

Python Prog + Input

$\rightsquigarrow$ string Σ_{ASCII}

Say $\sigma_0 \in \Sigma_{ASCII}$, e.g. $\sigma_0 = \langle BELL \rangle$

Given p, i two ASCII strings,

$$\langle p, i \rangle \overset{\text{define}}{=} p \circ i \in \Sigma^*_{ASCII}$$

" description of p and i "

$\underline{\text{Thm}}$: If $f: S \to \text{Power}(S)$,

then

$$T = \{ s \in S \mid s \notin f(s) \}$$

is not in $\text{Image}(f)$, i.e. there is no $t \in S$ such that $f(t) = T$.

Special case:

$$\text{LanguageRecBy}: \Sigma^*_{ASCII} \longrightarrow \text{Power}(\Sigma^*_{ASCII})$$

So

$$\boxed{T = \{ t \in \Sigma^*_{ASCII} \mid t \notin \text{LanguageRecBy}(t) \}}$$

then T is not in the image of LanguageRecBy.

We say that $L \subset \Sigma_{ASCII}^{*}$ is recognizable if it is in the image of LanguageRecBy, unrecognizable if not.

Pf of Cantor's Thm: Say for some $t \in S$, $f(t) = T$. Then either $t \in T$, or $t \notin T$

Say that

$$T = \{ s \in S \mid s \notin f(s) \}$$

and

$\boxed{t \in T}$ then: $t \notin f(t)$

but $f(t) = T$, so $t \notin f(t) = T$

so $\boxed{t \notin T}$

Contradiction!

Similarly $\boxed{t \notin T}$ so $t \in f(t) = T$

so $\boxed{t \in T}$

So Cantor's Thm $\Rightarrow$

GROUCHO_MARX_SELF

$$\boxed{\{ p \in \Sigma^*_{ASCII} \mid p \notin \text{LanguageRecBy}(p) \}}$$

is not recognizable

(unrecognizable).

Say NON-ACCEPTANCE $\overset{def}{=}$

$$\{ p \sigma_0 i \mid i \notin \text{LanguageRecBy}(p) \}$$

$$\{ p \sigma_0 i \mid p \underbrace{\text{does not accept}}_{\text{does not return "yes"}} i \}$$

Thm: NON-ACCEPTANCE is unrecognizable.

Pf Say that it were, i.e. some algorithm recognizes NON-ACCEPTANCE.

Then:

given $P \in \Sigma^*_{ASCII}$,

write down $P \sigma_0 P$

feed into NON-ACCEPTANCE as a subroutine, then we know if $P \notin \text{LanguageRecBy}(P)$, solved GROUCHO MARX ETC