We defined

\[ \text{LanguageRecBy} : \Sigma_{\text{ASCII}}^* \to \text{Power}(\Sigma_{\text{ASCII}}^*) \]

\[ \text{GROUCHO-MARX-SELF} = \]

\[ T = \{ p \in \Sigma_{\text{ASCII}}^* \mid p \notin \text{LanguageRecBy}(p) \} \]

is not recognizable, i.e. not in the image of LanguageRecBy(p).

Today: - Why? (Proof?)

- So what?
Answer to "So What?"

involves "reductions"

\[
\text{VALID-PYTHON-PROGRAMS} \\
\quad \subseteq \sum_{\text{ASCII}^*} \\
\text{We will talk about} \\
\text{Python Prog + Input} \\
\quad \mapsto \text{string } \sum_{\text{ASCII}^*} \\
\text{Say } \sigma_0 \in \sum_{\text{ASCII}^*}, \text{ e.g. } \sigma_0 = \langle \text{BELL} \rangle
\]
Given $p, i$ two ASCII strings,

$$\langle p, i \rangle \overset{\text{def}}{=} p \circ_i e \in \Sigma_{\text{ASCII}}$$
Thm: If \( f : S \to \text{Power}(S) \), then
\[ T = \{ s \in S \mid s \notin f(s) \} \]
is not in \( \text{Image}(f) \), i.e., there is no \( t \in S \) such that \( f(t) = T \).

Special case:

LanguageRecBy: \( \Sigma^{*} \rightarrow \text{Power}(\Sigma^{*}) \)

So
\[ \mathcal{F} = \{ t \in \Sigma^{*} \mid t \notin \text{LanguageRecBy}(t) \} \]
then $T$ is not in the image of $\text{LanguageRecBy}$.

We say that $L \subseteq \Sigma^{*}$ is recognizable if it is in the image of $\text{LanguageRecBy}$,

unrecognizable if not.

Pf of Carter's Thm: Say for some $t \in S$, $f(t) = T$. Then either $t \in T$, or $t \notin T$. 
Say that
\[ T = \{ s \in S \mid s \notin f(s) \} \]
and
\[ t \in T \] then: \( t \notin f(t) \)

but \( f(t) = T \), so \( t \notin f(t) = T \)

so \( t \notin T \)

Contradiction!

Similarly \( t \notin T \) so \( t \in f(t) = T \)

so \( t \in T \)
So Cantor’s Thm \[ \Rightarrow \]
\[
\{ p \in \Sigma^* \mid p \notin \text{Language}(B_p) \}
\]
is not recognizable (unrecognizable).

Say \text{NON-ACCEPTANCE} \( \overset{\text{def}}{=} \)
\[
\{ p \circ_0 i \mid i \notin \text{Language}(B_p) \}
\]
\[
\{ p \circ_0 i \mid p \text{ does not accept } i \}
\]
does not return “yes”
Theorem: NON-ACCEPTANCE is unrecognizable.

Proof: Say that it were, i.e., some algorithm recognizes NON-ACCEPTANCE.

Then:

given $p \in \Sigma^*$,

write down $p \overline{p}$

feed into NON-ACCEPTANCE

as a subroutine, then we know if $p \in \text{LanguageRecy}(\overline{p})$, solved GROUCHO MARX, etc.