CPSC 421/501 Sept 13, 2023

Today:

$$\text{LanguageRecBy} : \Sigma_{\text{ASCII}}^* \to \text{Power}(\Sigma_{\text{ASCII}}^*)$$

$$\begin{pmatrix} \text{Python} \\ \text{Algorithm} \end{pmatrix} \longmapsto \begin{pmatrix} \text{Language} \\ \text{recognized} \\ \text{by the algorithm} \end{pmatrix}$$

$$p \longmapsto \text{LanguageRecBy}(p)$$

Using this + Cantor's Theorem we build a language that is unrecognizable, i.e. not recognized by any algorithm.

Homework:

You can (1) use LaTeX

(2) write clearly

Illegible homework will not be graded, such as this type of writing

=

USE CANVAS TO GET TO GRADSCOPE

"I don't want to belong to any club that would accept me as one of its members."

Groucho Marx

"Python" with some conventions

later §4.2, (Ch 3) [Sip]

"Turing machine" with some conventions

"algorithm"

Conventions:

- one input, string in Σ_{ASCII}^*
- two return values "yes", "no"
- any other program behaviour we call "loops"

Fix some conventions:

In Σ_{ASCII}^*, some strings are in

VALID-PYTHON-PROGRAMS $\subset \Sigma_{ASCII}^*$

If $p \in$ VALID-P-P, then

on input $\underbrace{i \in \Sigma_{ASCII}^*}_{\text{a finite length } \Sigma_{ASCII} \text{ string}}$

We say

$\boxed{p \text{ accepts input } i}$ means on input i,

algorithm p reaches

" return("yes")

"p accepts i" means: on input i,

p reaches return("yes") ①

"p rejects i" means: on input i,

p reaches return("no") ②

"p loops on i" means: neither

① nor ②.

For any $p \in$ VALID-P-P,

LanguageRecognizedBy(p)

$\stackrel{\text{def}}{=} \{ i \in \Sigma^*_{ASCII} \mid p \text{ accepts } i \}$

LanguageRecBy(p) is therefore a subset of Σ^*_{ASCII}, i.e. an element of $Power(\Sigma^*_{ASCII})$.

If $p \notin$ VALID-PYTHON-PROGRAM, set $LanguageRecBy(p) \stackrel{def}{=} \emptyset$

So:

$$\boxed{\text{LanguageRecBy}: \quad \Sigma^*_{ASCII} \longrightarrow Power(\Sigma^*_{ASCII})}$$

E.g.

LanguageRecBy(
```
# This is a comment
i = input
if ( len(i) > 5 ):
    return("yes")
```
)

$$= \{ i \in \Sigma^*_{ASCII} \mid \text{length}(i) > 5 \}$$

$$(\#, \sqcup, T, h, i, s, \ldots, \langle CR \rangle, \ldots, y, e, s, ",), \langle CR \rangle) \in \Sigma^{45}_{ASCII}$$

LanguageRecBy(

\# _ _

\# _ _

i = input

⋮

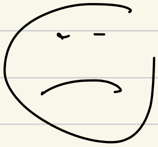
return(i+1)

)

never says "yes" = $\emptyset$

=

Decision Problems = yes 

no 

The answer "yes" gives us a language

$$\boxed{\text{LanguageRecBy}\left(\begin{array}{c} \text{"} \quad - \quad - \\ - \quad - \quad - \\ - \quad - \end{array}\right)} = \emptyset$$

any p that does not contain return("yes") somewhere

We say that "p halts on input i" if p on input i accepts or rejects, but does not loop i.e,

on input i, p either

- returns "yes"

OR

- " "no"

=

p is a <u>decider</u> if <u>on</u> $\left\{\begin{matrix}\text{all}\\ \underline{\text{any}}\end{matrix}\right\}$ input i, p either accepts or rejects (but does not loop)

LanguageRecBy :

$$\Sigma_{ASCII}^{*} \longrightarrow Power(\Sigma_{ASCII}^{*}),$$

i.e. $\forall p \in \Sigma_{ASCII}^{*}$,

LanguageRecBy(p) is some language over Σ_{ASCII}

We say $L \subset \Sigma_{ASCII}^{*}$

$$L \in Power(\Sigma_{ASCII}^{*})$$

is <u>recognizable</u> if

$L = \text{LanguageRecBy}(p)$

for some $p \in \Sigma^*_{ASCII}$

$=$

Cantor's Theorem $\Rightarrow$

$T = \{ \boxed{p \in \Sigma^*_{ASCII}} \mid p \notin \text{LangRecBy}(p) \}$

all

is not in the image of

LanguageRecBy !!!!

GROUCHO-MARX-SELF

$$p = \underbrace{\text{return("yes")}} \in \Sigma^*_{ASCII}$$

$$\text{LangRecBy}(\quad\quad) = \Sigma^*_{ASCII}$$