Using this + Cantor's Theorem we build a language that is unrecognizable, i.e. not recognized by any algorithm.
Homework:

You can (1) use LaTeX
(2) write clearly

Illegible homework will not be graded, such as this type of writing

USE CANVAS TO GET TO GRADSCOPE

"I don't want to belong to any club that would accept me as one of its members."

Groucho Marx
"Pyther" with some conventions
later §4.2, (Ch 3) [Sip]

"Turing machines" with some conventions
by "algorithm".

Conventions:
- one input, string in $\Sigma^*$
- two return values "yes", "no"
- any other program behaviour
  we call "loops"
Fix some conventions:

In $\Sigma^{\star}_{\text{ASCII}}$, some strings are in $\text{VALID-PYTHON-PROGRAMS} \subseteq \Sigma^{\star}_{\text{ASCII}}$

If $p \in \text{VALID-P-P}$, then

on input $i \in \Sigma^{\star}_{\text{ASCII}}$

a finite length $\Sigma^{\star}_{\text{ASCII}}$ string

We say $p$ accepts input $i$ means on input $i$, algorithm $p$ reaches "return ("yes")"
"p accepts i" means: on input i, p reaches return("yes")

"p rejects i" means: on input i, p reaches return("no")

"p loops on i" means: neither

1. nor 2.

For any \( p \in \text{VALID-P-P}, \)

Language Recognized By \( (p) \)

\[ \text{def} = \{ i \in \Sigma^* \mid \text{ASCII} \rightarrow \ p \text{ accepts } i \} \]
LanguageRecBy(p) is therefore a subset of $\Sigma^*$, i.e. an element of $\text{Power}(\Sigma_{\text{ASCII}}^*)$.

Set LanguageRecBy(p) := $\emptyset$

So:

LanguageRecBy:

$$\Sigma^* \rightarrow \text{Power}(\Sigma_{\text{ASCII}}^*)$$
E.g.,

```
LanguageRecBy(
    # This is a comment
    i = input
    if (len(i) > 5):
        return("yes")
)
```

\[
\{ i \in \sum_{\text{ASCII}} \mid \text{length}(i) > 5 \}
\]

\[
(\#, \n, \tt, \t, \h, \i, \s, \ldots, \langle \text{CR} \rangle, \ldots, \&;\, e, \s, \n, \t, \h, \i, \s, \ldots, \langle \text{CR} \rangle) \in \sum_{\text{ASCII}}^{45}
\]
Language Recognition

\[
\text{Language Recognize}(L, x) = \begin{cases} 
    \text{true}, & \text{if } x \in L \\
    \text{false}, & \text{otherwise}
\end{cases}
\]

\[
\text{return } (x \in L)
\]

\[
\text{neversays} \quad \Rightarrow \quad \emptyset
\]

\[
\text{Decision Problems} = \begin{cases} 
    \text{yes}, & \text{if } \text{true} \\
    \text{no}, & \text{otherwise}
\end{cases}
\]

The answer gives us "yes" and a language
LanguageRecBy \( \{ \) 

any \( p \) 

that does not contain 

return ("yes") 

somewhere

We say that 

"p halts on input i" 

if \( p \) on input \( i \) accepts or rejects, but does not loop
i.e., on input $i$, $p$ either
- returns "yes"
- or
- "no"

$\implies p$ is a decider if on
(\text{any} \text{ input}) \ p \ \text{either}
accepts or rejects (but
does not loop)
LanguageRecBy : $\Sigma^* \rightarrow \text{Power}(\Sigma_{\text{ASCII}}^*)$,

i.e. $\forall p \in \Sigma^*_{\text{ASCII}}$,

LanguageRecBy(p) is some language over $\Sigma_{\text{ASCII}}^*$

We say $L \subseteq \Sigma^*_{\text{ASCII}}$

$L \in \text{Power}(\Sigma^*_{\text{ASCII}})$

is recognizable if
\[ L = \text{LanguageRecBy}(p) \]

for some \( p \in \Sigma^* \)

\[ = \]

Cantor's Theorem \[ \Rightarrow \]

\[ T = \{ p \in \Sigma^*_\text{ASCII} \mid p \notin \text{LanguageRecBy}(p) \} \]

is not in the image of \( \text{LanguageRecBy} \) !!!

GROUCHO-MARX-SELF
\[ p = \text{return} \left( \text{"yes"} \right) \in \sum_{E \in \text{ASCII}}^{*} \]

\[ \text{LangRecBy} \left( \right) = \sum_{E \in \text{ASCII}}^{*} \]