CPSC 421/501 Sept. 11, 2023

"I don't want to belong to any club that would accept me as one of its members."

Groucho Marx

1890 – 1977

[Said to have been written by Groucho to explain his resignation from a club.]

Admin —

✓ Gradescope & Waitlist

✓ Corrections to HW 1 over weekend (due Thursday, 11:59pm)

✓ CS Website had issues over the weekend

— Other?

<u>Cantor 1:</u> $|S| < |Power(S)|$.

<u>Cantor 1':</u> Given $f : S \to Power(S)$, f is not surjective...

<u>Cantor 1'':</u> ... moreover $T = \{ s \in S \mid s \notin f(s) \}$ not in $Image(f)$

Back to alphabets, strings, ...

Alphabet is a finite, non-empty set.

If Σ is an alphabet,

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \ldots$$

$$\Sigma^k = \underbrace{\Sigma \times \ldots \times \Sigma}_{k \text{ copies}}$$

If $\Sigma = \{a, b\}$

$$\Sigma^* = \{ \varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots$$

$$ab = (a, b)$$

$\Sigma^* =$ set of strings over Σ.

A $\underline{\text{language over } \Sigma}$ is a subset of Σ^*, i.e. an element of Power(Σ^*).

E.g.

PALINDROME over $\Sigma = \{a, b, c\}$

$$\mathrm{PAL}_\Sigma \stackrel{\text{def}}{=} \{ w \in \Sigma^* \mid w^{rev} = w \}$$

where if $w = \sigma_1 \ldots \sigma_k$, $\sigma_i \in \Sigma$

then

$$w^{rev} = \sigma_k \sigma_{k-1} \ldots \sigma_2 \sigma_1$$

$$PAL_{\{a,b,c\}} = \{ \varepsilon, a, b, c, aa, bb, cc,$$
$$aaa, aba, aca, bab, bbb, \ldots,$$
$$abba, \ldots, abcccba, \ldots \}$$

$$\Sigma^k \longleftrightarrow \{ \text{Maps } [k] \to \Sigma \}$$

$$= \{ \text{Maps } \{1, 2, \ldots, k\} \to \Sigma \}$$

therefore ---

$$\Sigma^0 = \{ \text{Maps } \emptyset \to \Sigma \} = \{ \varepsilon \}$$

$\text{STARTS\_WITH\_a}_{\{a,b,c\}}$

$= \{a, aa, ab, ac, aaa, aab, \ldots\}$

$$\boxed{\Sigma_{digits} = \{0, 1, 2, \ldots, 9\}}$$

To $421 \in \mathbb{N} = \{1, 2, \ldots\}$

we associate string

$(4,2,1) \in \Sigma^3_{digits}$

$\quad \shortparallel$

$\langle 421 \rangle$

$\langle \ \rangle$ = "the description of"

$$\text{PRIMES} \subset \Sigma^*_{\text{digits}}$$

$$\|$$

$$\{2, 3, 5, 7, 11, 13, 17, \ldots\}$$

$$\subset \Sigma^*_{\text{digits}}$$

$\downarrow$ really

$$\{\langle 2\rangle, \langle 3\rangle, \langle 5\rangle, \ldots\}$$

$$\text{DIV-BY-7} = \left\{ \begin{array}{l} 7\,?\quad 00\,? \\ 0\,? \\ \varepsilon\,? \end{array} \right.$$

$$\subset \Sigma^*_{\text{digits}}$$

DIV-BY-7

$= \{0, 7, 14, 21, \ldots\}$

DIV-BY-7-WITH-LEADING-ZEROES-OK

$\{0, 7, 00, 07, 14, \ldots\}$

$\vdots$

$=$

$\Sigma_{bin} = \{0, 1\}$

$\langle 2 \rangle_{bin} = 10$

$\langle 3 \rangle_{bin} = 11$

$\vdots$

$\langle 7 \rangle_{\Sigma_{digits}} = 7$

$\langle 7 \rangle_{french} = \text{sept}$

$\langle 7 \rangle_{bin} = 111$

in CPSC 421, [Sip], ...

Problems, algorithms:

$$\{\text{Problem over } \Sigma\} \stackrel{\text{def}}{=} \text{Power}(\Sigma^*)$$

Def: A problem over Σ is a subset of Σ^*, i.e. a language over Σ

e.g. $\text{PALINDROME}_{\{a,b,c\}}$ "problem"

"decision problems"

e.g. PRIMES, DIV-BY-7, ...

A $\underline{\text{algorithm}}$ is ...

for us (before defining a Turing machine) for now, is a Python program (or C, C++, ...)

meaning...

$$\Sigma_{ASCII} = \text{size } 256$$

$$= \{ \ldots, a, b, \ldots, z, \\ A, B, \ldots, Z, \\ 0, 1, \ldots, 9, \\ ;, ,, \ldots, \langle CR \rangle, \ldots \}$$

We need:

input statement : run once

"same functionality" (see § 4.2 of [Sip])

output statement
return "

— "yes"
other stuff

Our program:

$$(\#, \langle CR \rangle, \#, \sqcup, p, y, t, h, o, n, \dots)$$

$$\in \Sigma^*_{ASCII}$$

$$\left\{ \begin{matrix} \text{Valid Python} \\ \text{programs} \end{matrix} \right\} \subset \Sigma^*_{ASCII}$$

For each $p \in \left\{ \begin{matrix} \text{Valid Python} \\ \text{program} \end{matrix} \right\}$

given input i $\left\{ \begin{matrix} \text{"}p \text{ accepts } i\text{"} \\ \text{if on input } i, \\ p \text{ reaches} \\ \text{return("yes")} \end{matrix} \right.$

For each such "p",

$$\text{LanguageRecBy}(p) = \{ i \in \Sigma^*_{ASCII} \text{ s.t. } p \text{ accepts } i \}$$