"I don't want to belong to any club that would accept me as one of its members."

Groucho Marx
1890 - 1977

[Said to have been written by Groucho to explain his resignation from a club.]
Admin -

✓ Gradescope & Waitlist

✓ Corrections to HW 1 over weekend (due Thursday, 11:59 pm)

✓ CS Website had issues over the weekend

- Other?

Cantor 1: \[|S| < |\text{Power}(S)|.\]

Cantor 1': Given \(f: S \rightarrow \text{Power}(S)\),
\(f\) is not surjective...

Cantor 1'': ...moreover \(\overline{T} = \{S \in S \mid s \notin f(s)\}\) not in \(\text{Image}(f)\)
Back to alphabets, strings, -

**Alphabet** is a finite, non-empty set.

If \( \Sigma \) is an alphabet,

\[
\Sigma^* = \Sigma^0 \cup \Sigma \cup \Sigma^2 \cup \Sigma^3 \cup \ldots
\]

\[
\Sigma^k = \underbrace{\Sigma \times \ldots \times \Sigma}_{k \text{ copies}}
\]

If \( \Sigma = \{a, b\} \)

\[
\Sigma^* = \{ \varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots \}
\]

\[
ab = (a, b)
\]
$\Sigma^* = \text{set of strings over } \Sigma$.

A language over $\Sigma$ is a subset of $\Sigma^*$, i.e. an element of $\text{Power}(\Sigma^*)$.

E.g.

$\text{PALINDROME over } \Sigma = \{a, b, c\}$

$\text{PAL}_\Sigma \overset{\text{def}}{=} \{ w \in \Sigma^* \mid \text{rev}(w) = w \}$

where if $w = \sigma_1 \ldots \sigma_k$, $\sigma_i \in \Sigma$ then

$\text{w rev} = \sigma_k \sigma_{k-1} \ldots \sigma_2 \sigma_1$
\( \text{PAL}_{\{a,b,c\}} = \{ \varepsilon, a, b, c, aa, bb, cc, aaa, aba, aca, bab, bbb, \ldots, abba, \ldots, abc\, c\, ba, \ldots \} \)

\[ \Sigma^k \leftrightarrow \{ \text{Maps} \{1,2,\ldots,k\} \rightarrow \varepsilon \} \]

\[ \circ \Sigma = \{ \text{Maps} \emptyset \rightarrow \varepsilon \} = \{ \varepsilon \} \]
STARTS WITH a \{a, b, c\}

= \{a, ac, ab, ac, aac, ab\, \ldots \}

\sum_{\text{digits}} = \{0, 1, 2, \ldots, 9\}

To \(421 \in \mathbb{N} = \{1, 2, \ldots\}\) we associate string

\((4, 2, 1) \in \sum^3_{\text{digits}} \)

\langle 421 \rangle

\langle \rangle = "the description of"
PRIMES \subseteq \Sigma^*_{\text{digits}}

\{ 2, 3, 5, 7, 11, 13, 17, \ldots \}

\downarrow \text{really}

\{ \langle 2 \rangle, \langle 3 \rangle, \langle 5 \rangle, \ldots \}

DIV-BY-7 = \{ 7, 00, \text{?} \}

\subseteq \Sigma^*_{\text{digits}}
DIV-BY-7
= \{ 0, 7, 14, 21, ... \}

DIV-BY-7 - WITH-LEADING-ZEROS - OK
= \{ 0, 7, 00, 07, 14, ... \}

Σ₁bin = \{ 0, 1 \}

⟨2⟩bin = 10

⟨3⟩bin = 11

⟨7⟩bin = 111

⟨7⟩₁₀ = 7

⟨7⟩₇₇₇ = sept

⟨7⟩₁₇₇₇ = sept
Problems, algorithms:

\[
\text{def } = \text{ Power}(\Sigma^*)
\]

Def: A problem over \( \Sigma \) is a subset of \( \Sigma^* \), i.e. a language over \( \Sigma \).

E.g. PALINDROME \{a,b,c\} problem

"decision problems"

E.g. PRIMES, DIVBY7, ...
A algorithm is ---
for us (before defining a Turing machine) for now, is a Python program (or C, C++, ...)
meaning ---

\[ \sum_{\text{ASCII}} = \text{size 256} \]

\[ = \{ \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , \_\_\_\_ , 0 , 1 , \ldots , 9 , A , B , \ldots , Z , a , b , \ldots , z , \_\_\_\_ , \_\_\_\_ \} \]
We need:

```
input statement:  run once

"same functionality" (see § 4.2 of [Sip])
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output statement

{ return "

- { "yes"

- { other stuff

- Our program:

( #, <CR>, #, \ll, \ll, p, y, t, h, o, n, ... )

\in \Sigma_k^{ASCII}
```
\[
\{ \text{Valid Python} \} \subseteq \Sigma^+
\]

For each \( p \in \{ \text{Valid Python} \} \) program,

given input \( i \) \( p \) accepts if on input \( i \),
\( p \) reaches
\( \text{return(“yes”) \} \)

For each such \( p \),

\( \text{LanguageRecBy}(p) = \{ \ i \in \Sigma^* \text{ASCII} \ |
\text{s.t. } p \text{ accepts } i \} \)