CPSC 421/501 2023 Homework 4 Solutions

7.3.2 (b)

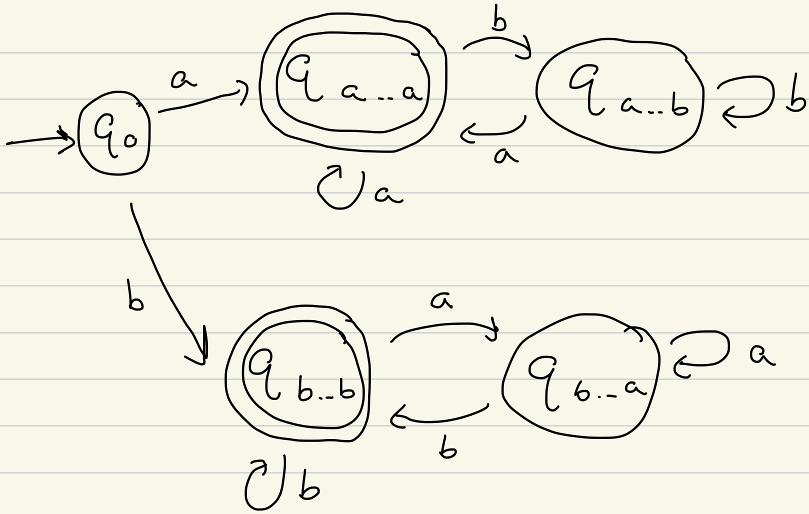
If Geddy blames themself: then Geddy is not a person who blames themself. Hence Geddy is not blamed by Geddy. Hence Geddy is not blamed by themself, a contradiction.

If Geddy is not blamed by themself: then Geddy is a person who blames themself. Hence Geddy is blamed by Geddy. Hence Geddy is blamed by themself, a contradiction.

(2) The algorithm: from the initial state, q_0, transition to a group of 2 states, say $q_{a..a}$ and $q_{a..b}$, according to whether the last symbol read is, respectively, "a" or "b". If the input ends when are in state $q_{a..a}$, then the first and last symbol are the same (both "a"), so the input lies in L and so $q_{a..a}$ is an accepting state. Similarly $q_{a..b}$ is a rejecting state.

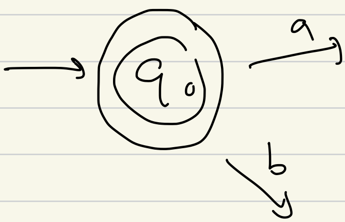
Similarly if the first symbol is a "b", with a group of 2 other states, say $q_{b..b}$ and $q_{b..a}$.

Hence a DFA that recognizes L is



Note: You can make q_0 accepting if you like, since it is unclear if $\varepsilon \in L$ or $\varepsilon \notin L$.

So



etc.

is also OK.

3(a) Say that $n = 7p + a$ and $m = 7q + b$

for some $a, b \in \{0, 1, .., 6\}$ and integers p, q.

Then

$$10m + n = 10(7q + b) + 7p + a$$

$$= 7(10q + b + p) + 3b + a$$

Hence

$$(10m + n) - (3b + a) \text{ is divisible by } 7,$$

so

$$(10m + n) \bmod 7 = (3b + a) \bmod 7.$$

(b) Let $Q = \{q_0, q_0', q_1', .., q_7'\}$, where

q_0 is the initial state (which we reject, since $\varepsilon \notin L$)

and for $i = 0, 1, ..., 6$, q_i' means that if w is

the input seen until this point, then $w \bmod 7 = i$

(when $w \neq \varepsilon$) (and w may have leading zeros,

Then

$$\delta(q_0, i) = q'_{i \bmod 7} \qquad (i = 0, 1, \ldots, 9)$$

takes us to the correct state upon reading the first input symbol, and by part (a), for $a = 0, 1, \ldots, 6$ and $n = 0, 1, \ldots, 9$, the formula

$$\delta(q'_a, n) = q'_{(3a+n) \bmod 7}$$

takes an input of the form $w\sigma$ where $w \bmod 7 = a$ and $\sigma \in \{0, 1, \ldots, 9\}$ and transitions to the correct value of $w\sigma \bmod 7$. Hence

$Q = \{q_0, q'_0, q'_1, \ldots, q'_6\}$, Σ, δ as above

initial state q_0, and $F = \{q'_0\}$

is such a DFA (i.e. the input is divisible by 7 iff the input is ~~tak~~ taken to q'_0).

(c) (i) it suffices to make q_0 accepting, i.e. take $F = \{q_0, q_0'\}$, since the DFA is taken to state q_0 by an input iff the input equals ε.

(ii) One can "merge" q_0 and q_0', that is let $Q = \{q_0', q_1', \ldots, q_6'\}$ with q_0' the initial state, q_0' the only accepting state, and to restrict δ as above to states $q_0', \ldots, q_6'$, i.e.

$$\delta(q_a', n) = q'_{(3a+n) \bmod 7}$$

4 (a) Similarly to Problem (3), we have

$$(2m+n) \bmod 3 = \big(2(m \bmod 3) + (n \bmod 3)\big) \bmod 3.$$

Hence, similarly to Problem (3), we can take

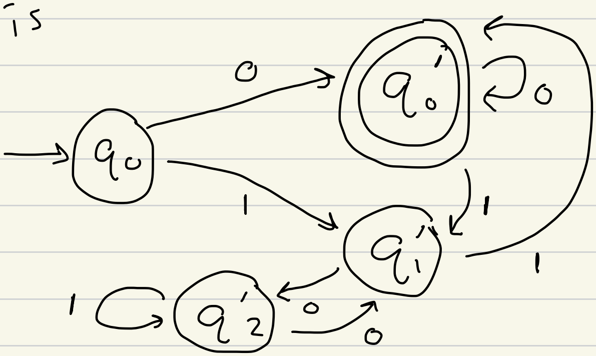
$Q = \{q_0, q_0', q_1', q_2'\}$, $F = \{q_0'\}$ (the sole accepting state)

q_0 the initial state, and

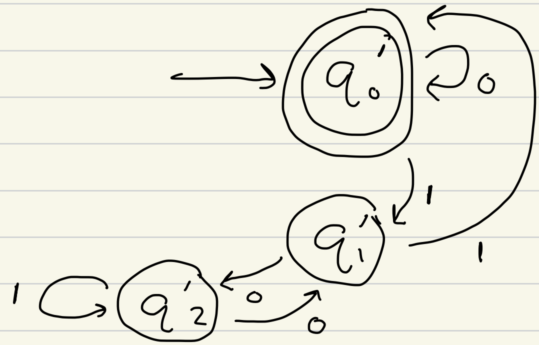
$$\delta(q_0, n) = q'_{n \bmod 3}$$

$$\delta(q'_m, n) = q'_{(2m+n) \bmod 3}.$$

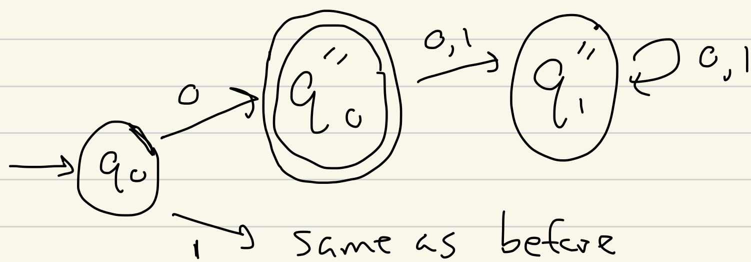
The depiction of this DFA as a labelled graph is



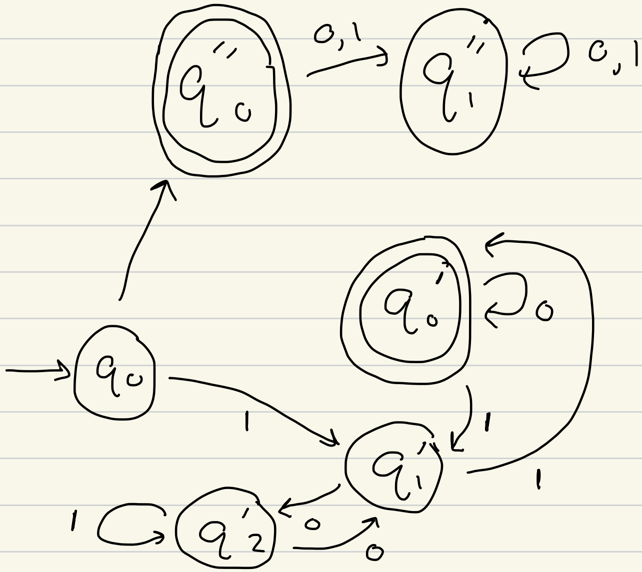
4(b) Similarly to Problem (3), we can now "merge q_0 and q_0'" giving



(c) we must now reject any string beginning with a 0 expect 0 itself. Hence we take the DFA in (a) and change the transition from q_0 to q_0' to a separate "branch" of the DFA:



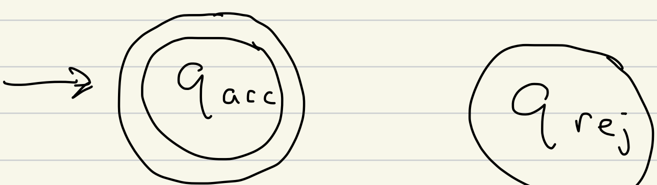
hence the complete DFA is therefore



(5) Let $(Q, \Sigma_{bits}, \delta, q_0, F)$ be a DFA recognizing L'.

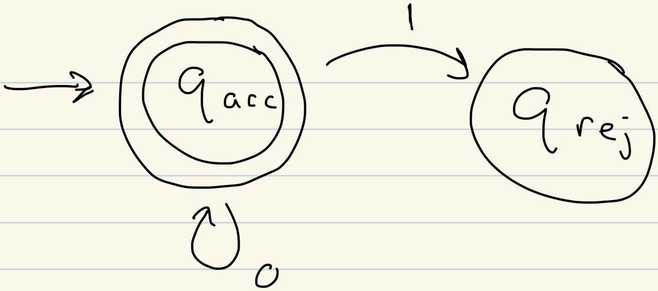
Since some elements of Σ_{bits}^* lie in L', i.e. $L' \neq \phi$, Q must contain an accepting state. Similarly, since $L' \neq \Sigma_{bits}^*$, Q must contain a rejecting state.

If $|Q| = 2$, then $Q = \{q_{acc}, q_{rej}\}$ where q_{acc} is accepting and q_{rej} is rejecting. Since $\varepsilon \in L$, q_{acc} is also the initial state; in other words, the DFA so far looks like:

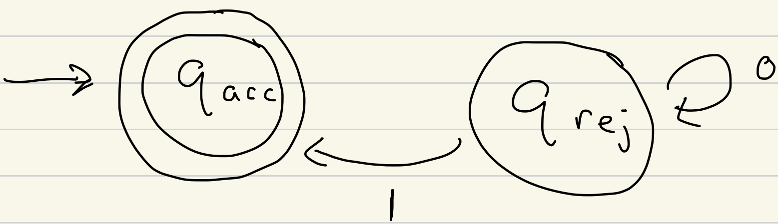


(but we don't know the transitions)

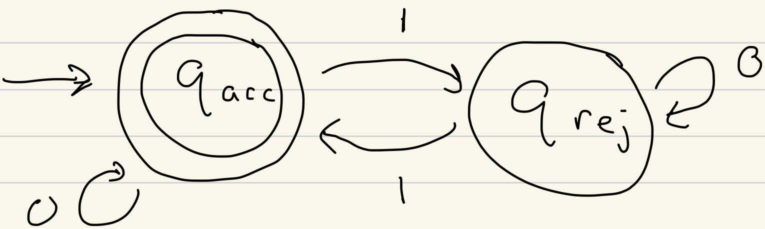
since $0 \in L'$ and $1 \notin L'$, the arrows from q_{acc}, the initial state, must be



Hence the input $1w$, for any $w \in \Sigma_{bits}^*$, is taken to q_{rej} after reading the symbol 1. Since $10 \notin L'$ and $11 \in L'$, the arrows from there must be



Hence the DFA must be



But this DFA accepts the string 101, which (is binary for 5 and) does not lie in L'. Hence this DFA does not recognize L'.

Hence our assumption that some DFA with 2 states (or fewer) recognizes L' gives us a contradiction.

[Remark: This kind of reasoning will be made systematic when we learn the Myhill-Nerode theorem.]