7.2.2 Let \( f(1) = f(2) = f(3) = \{1, 2, 3\} \). Then for all \( S = 1, 2, 3, \ S \in \{1, 2, 3\} = f(S) \). Hence
\[
T = \{ S \in \{1, 2, 3\} \mid S \in f(S) \} = \{1, 2, 3\},
\]
and \( \{1, 2, 3\} \) lies in the image of \( f \).

[There are many other examples, e.g. \( f(1) = f(2) = f(3) = \emptyset \)...]

7.2.3 (a) Since \( 1 \in \{1, 2\} = f(1) \), it is not true that \( 1 \notin f(1) \) and hence \( 1 \notin T \). Since \( 1 \in f(1) \) and \( 1 \notin T \), \( T \notin f(1) \).

(b) If \( f(2) = \emptyset \), then \( 2 \notin \emptyset = f(2) \) and so \( 2 \notin T \). On the other hand, if \( f(2) = \{1, 2, 3\} \), then \( 2 \in \{1, 2, 3\} = f(2) \), so \( 2 \notin T \). Hence both \( 2 \in T \) and \( 2 \notin T \) are possible.
7.2.4.

\[ n \in f(n) \iff n + \frac{n}{2} = k^2 \text{ for some } k \in \mathbb{N} \]

\[ \iff \frac{3n}{2} = k^2 \text{ for some } k \in \mathbb{N} \]

\[ \iff n = \frac{2k^2}{3} \text{ for some } k \in \mathbb{N} \]

Since \( \frac{2k^2}{3} \) is an integer, the prime factorization of \( k^2 \) must include 3 as a factor, and hence so does \( k \). Hence \( k = 3a \) for some \( a \in \mathbb{N} \), and hence \( n = 6a^2 \).

So \( n \in f(n) \iff n \in \{ 6a^2 \mid a \in \mathbb{N} \} = \{ 6, 6 \cdot 4, 6 \cdot 9, \ldots \} \),

So \( n \notin f(n) \iff n \in \mathbb{N} \setminus \{ 6, 6 \cdot 4, 6 \cdot 9, \ldots \} \)

So \( T = \mathbb{N} \setminus \{ 6, 6 \cdot 4, 6 \cdot 9, \ldots \} \)

\[ = \mathbb{N} \setminus \{ 6a^2 \mid a \in \mathbb{N} \} \]

\[ = \{ n \in \mathbb{N} \mid n \text{ is not } 6 \text{ times a perfect square} \} \]
If $t \notin T$, then similarly $t$ does not satisfy $t \notin f(g(t))$, so $t \notin f(g(t)) = f(t') = T$, so $t \notin T$, contradicting the assumption that $t \notin T$. 

If $t \in T$, then $s \in S \mid t \notin f(g(t)) \} \) then $t \notin f(g(t))$. But $g(t') = t'$, so $t \notin f(t')$. But $f(t') = T$, so $t \notin T$. This contradicts the assumption that $t \in T$. 

Either $t \in T$ or $t \notin T$. 

7.2.13. (a) \( \forall r \in R, \ r \text{ loves themself}, \) so \( r \notin T. \) Hence \( T = \emptyset = \{ \}. \)

(b) If David loves David (themself), then \( \text{David} \notin T. \) But also \( \text{David} \in \{ \text{people whom David loves} \}. \)

So \( T \neq \{ \text{people whom David loves} \}. \)