Homework 1, CPSC 421/501, Fall 2023

Problems: Exercises 7.2.5, 7.2.10, 7.2.23, 7.2.24

7.2.5

(a) $f(A) = \emptyset$, and

$f(C) = \{A, B, C\} = P$

(b) Since $A \notin f(A)$, $A \in T$

Since $C \in f(C)$, $C \notin T$

We know $T \neq f(A)$, since $A \notin f(A)$ but $A \in T$. We know $T \neq f(C)$ since $C \in f(C)$ but $C \notin T$.

(C) Now we know $f(B) = \{A, C\}$, and have $B \notin f(B)$ so $B \in T$.

Since $C \notin T$ and $A, B \in T$, we have $T = \{A, B\}$.

=

7.2.16

(a) $T = \{\text{David}\}$

(b) If David does not love themself, then $\text{David} \in T$ and $\{\text{People Whom David Loves}\} = \emptyset$.

So $\text{David} \notin \{\text{People Whom David Loves}\}$

but $\text{David} \in T$

so $T \neq \{\text{People Whom David Loves}\}$

7.2.23

(a) Oppenheimer $\in f(A)$, but we don't know which other movies lie in $f(A)$

(b) g defined by

$$g(\text{Oppenheimer}) = A$$
$$g(\text{Barbie}) = B$$
$$g(\text{Encounters}) = C$$
$$g(2001) = D$$

(c) Oppenheimer $\in f(A) = f(g(\text{Oppenheimer}))$

Barbie $\notin f(B) = f(g(\text{Barbie}))$

Encounters $\notin f(C) = f(g(\text{Encounters}))$

$2001 \in f(D) = f(g(2001))$

Hence

$$T = \{ s \mid s \notin f(g(s)) \}$$

$$= \{ \text{Barbie, Encounters} \}$$

We have:

$T \neq f(g(\text{Oppenheimer}))$, since

Oppenheimer $\in f(g(\text{Oppenheimer}))$ but

Oppenheimer $\notin T$.

Similarly $T \neq f(g(\text{Barbie}))$, since

Barbie $\notin f(g(\text{Barbie}))$ but

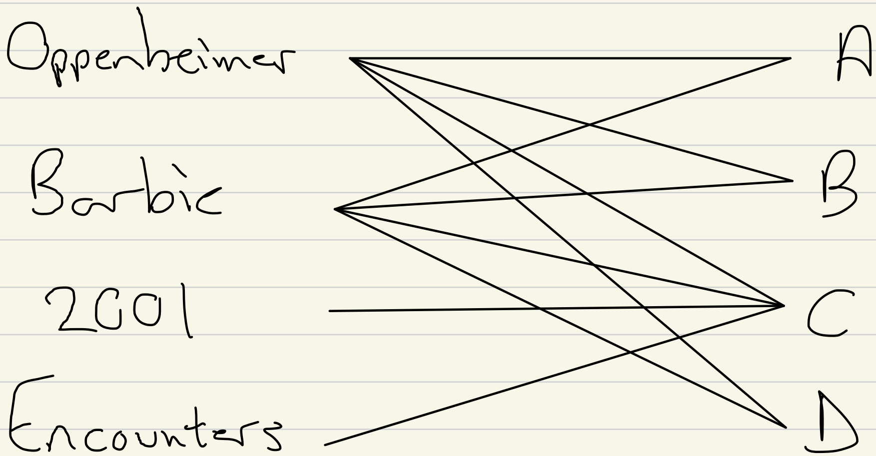
Barbie $\in T$

Similarly $T \neq f(g(s))$ for all $s \in S$.

Hence if T is the set of movies

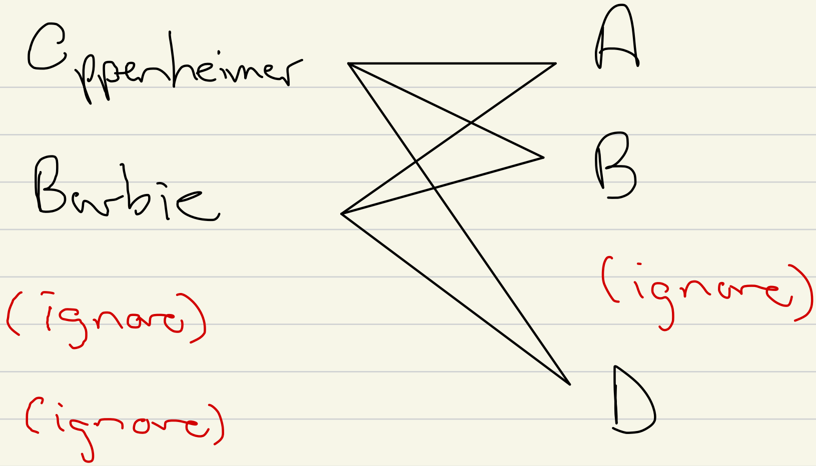
seen by x, then $x \notin \{A, B, C, D\}$.

7.2.24 We have the following information



Here a line (edge) means that we know if a movie was seen by a person — it doesn't matter if it was or wasn't.

For A, B, D there are only arrows from Oppenheimer and Barbie:



So any map $g: S \to S'$

that is built from this information

can only have $g(x) = A, B, D$

if $x =$ Oppenheimer, Barbie.

So one of A, B, D is not in

the image of g.

(Note that you don't really need

to draw the graph above, but it may help. This type of problem goes under the umbrella terms "matching" or "bipartite matching." If you have solved Sudoku puzzles, you have likely appealed to similar ideas.)