Problems: Exercises 7.2.5, 7.2.10, 7.2.23, 7.2.24

(a) \( f(A) = \emptyset \), and
\[ f(C) = \{A, B, C\} = P \]

(b) Since \( A \notin f(A) \), \( A \in T \)
Since \( C \in f(C) \), \( C \notin T \)
We know \( T \notin f(A) \), since \( A \notin f(A) \)
but \( A \notin T \). We know \( T \notin f(C) \) since
\( C \in f(C) \) but \( C \notin T \).
(c) Now we know $f(B) = \{A, C\}$, and have $B \notin f(B)$ so $B \notin T$.

Since $C \notin T$ and $A, B \in T$, we have $T = \{A, B\}$.

7.2.16

(a) $T = \{David\}$

(b) If David does not love themself, then $David \notin T$ and $\{People \, Whom \, David \, Loves\} = \emptyset$.

So $David \notin \{People \, Whom \, David \, Loves\}$ but $David \in T$

so $T \neq \{People \, Whom \, David \, Loves\}$. 
(a) Oppenheimer ∈ f(A), but we don’t know which other movies lie in f(A)

(b) g defined by

\[ g(\text{Oppenheimer}) = A \]
\[ g(\text{Barbie}) = B \]
\[ g(\text{Encounters}) = C \]
\[ g(2001) = D \]

(c) Oppenheimer ∈ f(A) =

\[ f(g(\text{Oppenheimer})) \]

Barbie ∈ f(B) = f(g(Barbie))

Encounters ∈ f(C) = f(g(Encounters))

2001 ∈ f(D) = f(g(2001))
Hence

\[ T = \{ s \mid s \in f(g(s)) \} \]

\[ = \{ \text{Barbie, Encounters} \} \]

We have:

\[ T \neq f(g(\text{Oppenheimer})) \text{, since} \]
\[ \text{Oppenheimer} \in f(g(\text{Oppenheimer})) \text{ but} \]
\[ \text{Oppenheimer} \notin T. \]

Similarly, \( T \neq f(g(\text{Barbie})) \), since

\[ \text{Barbie} \in f(g(\text{Barbie})) \text{ but} \]
\[ \text{Barbie} \notin T. \]

Similarly, \( T \neq f(g(s)) \) for all \( s \in S \).

Hence if \( T \) is the set of movies seen by \( x \), then \( x \notin \{ A, B, C, D \} \).
7.2.24 We have the following information:

Oppenheimer

Barbie

2001

Encounters

Here a line (edge) means that we know if a movie was seen by a person— it doesn’t matter if it was or wasn’t.

For A, B, D there are only arrows from Oppenheimer and Barbie.
So any map \( g : S \to S' \) that is built from this information can only have \( g(x) = A, B, D \) if \( x = \text{Oppenheimer}, \text{Barbie} \). So one of \( A, B, D \) is not in the image of \( g \).
(Note that you don’t really need
to draw the graph above, but it may help. This type of problem goes under the umbrella terms “matching” or “bipartite matching.” If you have solved Sudoku puzzles, you have likely appealed to similar ideas. )