GROUP HOMEWORK 9, CPSC 421/501, FALL 2023

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

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(0) Who are your group members? Please print if writing by hand.

(1) Let $L \in \mathrm{NP}$. Is L^* necessarily in NP? Explain.

(2) Let $L \in \mathrm{P}$. Is L^* necessarily in P? Explain. [Hint: if $1 \le a < b \le n$, then $\sigma_a \ldots \sigma_b \in L^*$ iff $\sigma_a \ldots \sigma_b \in L$ or for some $a \le c < b$ we have $\sigma_a \ldots \sigma_c \in L^*$ and $\sigma_{c+1} \ldots \sigma_b \in L^*$.]

(3) Let $n \ge 4$, and let $a_1, \ldots, a_n \in \{T, F\}$. Show that

$$a_1 \vee a_2 \vee \ldots \vee a_n = T$$

iff the formula

$$f(z_1, \ldots, z_{n-3}) = \bigl(a_1 \vee a_2 \vee z_1\bigr) \wedge \bigl(\neg z_1 \vee a_3 \vee z_2\bigr) \wedge \ldots \wedge \bigl(\neg z_{n-4} \vee a_{n-2} \vee z_{n-3}\bigr) \wedge \bigl(\neg z_{n-3} \vee a_{n-1} \vee a_n\bigr)$$

is satisfiable.

(4) Say that SAT $\in$ P. Give a polynomial time algorithm that given a satisfiable Boolean formula $f = f(x_1, \ldots, x_n)$ returns values $a_1, \ldots, a_n \in \{T, F\}$ such that $f(a_1, \ldots, a_n) = T$. [Hint: if f is satisfiable, then either $f(T, x_2, \ldots, x_n)$ is satisfiable or $f(F, x_2, \ldots, x_n)$ is satisfiable.]

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