GROUP HOMEWORK 9, CPSC 421/501, FALL 2023

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Please note:
(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but homework that is too difficult to read will not be graded.
(4) You may work together on homework in groups of up to four, but you must submit a single homework as a group submission under Gradescope.

(0) Who are your group members? Please print if writing by hand.

(1) Let \( L \in \text{NP} \). Is \( L^* \) necessarily in \( \text{NP} \)? Explain.

(2) Let \( L \in \text{P} \). Is \( L^* \) necessarily in \( \text{P} \)? Explain. [Hint: if \( 1 \leq a < b \leq n \), then \( \sigma_a \ldots \sigma_b \in L^* \) if \( \sigma_a \ldots \sigma_b \in L \) or for some \( a \leq c < b \) we have \( \sigma_a \ldots \sigma_c \in L^* \) and \( \sigma_{c+1} \ldots \sigma_b \in L^* \).]

(3) Let \( n \geq 4 \), and let \( a_1, \ldots, a_n \in \{T,F\} \). Show that
\[
a_1 \lor a_2 \lor \ldots \lor a_n = T
\]
iff the formula
\[
f(z_1, \ldots, z_{n-3}) = (a_1 \lor a_2 \lor z_1) \land (\neg z_1 \lor a_3 \lor z_2) \land \ldots \land (\neg z_{n-4} \lor a_{n-2} \lor z_{n-3}) \land (\neg z_{n-3} \lor a_{n-1} \lor a_n)
\]
is satisfiable.

(4) Say that \( \text{SAT} \in \text{P} \). Give a polynomial time algorithm that given a satisfiable Boolean formula \( f = f(x_1, \ldots, x_n) \) returns values \( a_1, \ldots, a_n \in \{T,F\} \) such that \( f(a_1, \ldots, a_n) = T \). [Hint: if \( f \) is satisfiable, then either \( f(T, x_2, \ldots, x_n) \) is satisfiable or \( f(F, x_2, \ldots, x_n) \) is satisfiable.]

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