# INDIVIDUAL HOMEWORK 7, CPSC 421/501, FALL 2023

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must write up your own solutions individually and must acknowledge with whom you worked.** You must also acknowledge any sources you have used beyond the textbook and two articles on the class website.

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**In the exercise below, for any $k \in \mathbb{N}$, let $\Sigma = \{a, b\}$, and let $C_k$ be,**

$$C_k = \bigl\{w \in \Sigma^* \mid \textbf{the } k\textbf{-th last symbol of } w \textbf{ is } a\bigr\}$$
$$= \bigl\{\sigma_1 \dots \sigma_m \mid \sigma_1, \dots, \sigma_m \in \Sigma,\ m \ge k\textbf{, and } \sigma_{m-k+1} = a\bigr\}.$$

**Hence an element of $C_k$ is of length at least $k$. For example,**

$$C_2 = \{aa, ab, aaa, aab, baa, bab, aaaa, aaab, \dots\}.$$

(1) For a language, $L$, over any alphabet, we define the *reverse of* $L$ to be the language
$$L^{\mathrm{rev}} = \{w^{\mathrm{rev}} \mid w \in L\},$$
where $w^{\mathrm{rev}}$ denotes the reverse word of $w$, i.e., for $w = \sigma_1 \dots \sigma_n$, $w^{\mathrm{rev}} = \sigma_n \dots \sigma_1$.
  (a) For any $k \in \mathbb{N}$, briefly describe an algorithm that could be implemented by a DFA to recognize $(C_k)^{\mathrm{rev}}$.
  (b) For any $k \in \mathbb{N}$, give a DFA with $k + 2$ states that implements the algorithm in part (a), and briefly explain why it does so.
  (c) For any $k \in \mathbb{N}$, briefly describe an algorithm that could be implemented by an NFA (not a DFA) to recognize $C_k$.

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(d) For any $k \in \mathbb{N}$, give an NFA with $k+1$ states that implements the algorithm in part (c), and briefly explain why it does so.

(2) Use the Myhill-Nerode Theorem to show that any DFA accepting

$$C_1 = \{w \in \Sigma^* \mid \text{the last symbol of } w \text{ is } a\}$$

must have at least two states.

(3) **The following problem will NOT be collected, but you should consider it good practice for the mideterm.** Use the Myhill-Nerode Theorem to show that any DFA accepting

$$C_1^{\mathrm{rev}} = \{w \in \Sigma^* \mid \text{the first symbol of } w \text{ is } a\}$$

must have at least three states. **The solution will NOT be released with Homework 7 solutions, but will be released before the midterm.**

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