GROUP HOMEWORK 7, CPSC 421/501, FALL 2023

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

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(1) For any alphabet, Σ, let

$$\mathrm{PALINDROME}_\Sigma = \{w \in \Sigma^* \mid w^{\mathrm{rev}} = w\},$$

where, as usual, if $w = \sigma_1 \ldots \sigma_k$ is a string of length k, w^{rev} denotes the reverse string of w, i.e.,

$$(\sigma_1 \ldots \sigma_k)^{\mathrm{rev}} = \sigma_k \ldots \sigma_1.$$

Let $\Sigma = \{a, b\}$, and

$$L = \mathrm{PALINDROME}_\Sigma = \{\epsilon, a, b, aa, bb, aaa, aba, bab, bbb, aaaa, abba, \ldots\}.$$

Show that L is non-regular using the Myhill-Nerode Theorem; in other words, give an infinite sequence of strings, $s_1, s_2, \ldots$ and prove that for all distinct $i, j \in \mathbb{N}$,

$$\mathrm{AccFut}_L(s_i) \neq \mathrm{AccFut}_L(s_j).$$

In all the exercises below, for any $k \in \mathbb{N}$, let $\Sigma = \{a, b\}$, and let C_k be,

$$C_k = \bigl\{w \in \Sigma^* \mid \textbf{the } k\textbf{-th last symbol of } w \textbf{ is } a\bigr\}$$
$$= \bigl\{\sigma_1 \ldots \sigma_m \mid \sigma_1, \ldots, \sigma_m \in \Sigma,\ m \geq k, \textbf{ and } \sigma_{m-k+1} = a\bigr\}.$$

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Hence an element of C_k is of length at least k. For example,

$$C_2 = \{aa, ab, aaa, aab, baa, bab, aaaa, aaab, \ldots\}.$$

(2) (a) Explain why

$$\mathrm{AccFut}_{C_3}(aaa) \neq \mathrm{AccFut}_{C_3}(baa),$$

by finding a string in $\mathrm{AccFut}_{C_3}(aaa)$ that does not lie in $\mathrm{AccFut}_{C_3}(baa)$.

(b) Similarly, show that for any symbols $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \in \Sigma$ we have

$$\mathrm{AccFut}_{C_3}(a\sigma_1\sigma_2) \neq \mathrm{AccFut}_{C_3}(b\sigma_3\sigma_4).$$

(c) Find a string, w, of length three over Σ such that

$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(a).$$

[Note: parts (c,d) are based on similar principles, and some students may find (d) easier than (c).]

(d) Find a string, w, of length three over Σ such that

$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(\epsilon).$$

(e) Using the Myhill-Nerode Theorem, and the previous parts, show that the minimum number of states of a DFA recognizing C_3 is 8. Do this by showing that (i) for distinct $w, w' \in \Sigma^3$, i.e., with $w \neq w'$, we have $\mathrm{AccFut}_{C_3}(w) \neq \mathrm{AccFut}_{C_3}(w')$, and (ii) for any string s, we have $\mathrm{AccFut}_{C_3}(s) = \mathrm{AccFut}_{C_3}(w)$ for some $w \in \Sigma^3$. **DO NOT** describe the transition function, δ, the initial state, q_0, or the set of final/accepting states, F.

(3) Consider a DFA, $M = (Q, \Sigma, \delta, q_0, F)$ with 4 states recognizing C_2, based on the ideas of the previous exercise and the Myhill-Nerode Theorem. (You should have one state for each string of length 2 over Σ.)

(a) Let q is the state corresponding to those strings whose accepting future set equals $\mathrm{AccFut}_{C_2}(bb)$. What is $\delta(q, a)$? Explain.

(b) Let q is the state corresponding to those strings whose accepting future set equals $\mathrm{AccFut}_{C_2}(w)$, where $w = \sigma_1\sigma_2$ is any string of length two. What are the values of $\delta(q, a)$ and $\delta(q, b)$? Explain.

(c) What is q_0, the intial state of M? Explain.

(d) What is F, the set of accepting/final states of M? Explain.

(e) Draw a diagram of the DFA (i.e., as usual, circles for the states, arrows for the δ function, etc.).

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