GROUP HOMEWORK 4, CPSC 421/501, FALL 2023

JOEL FRIEDMAN

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

---

(1) Exercise 7.3.2(b) on the handout "Uncomputability in CPSC421/501."

(2) Let $L \subset \{a,b\}^*$ be the language of strings whose first and last symbol are equal. Describe a DFA that recognizes L, and **explain why your DFA recognizes L (i.e., explain how your algorithm works)**.

(3) Let L be the set of strings of digits, i.e., strings over the alphabet $\Sigma_{\rm digits} = \{0,1,\ldots,9\}$, that represent integers in base 10 that are divisible by 7, where we allow leading 0's but we don't consider the empty string, ϵ, to be part of L. Hence

$$L = \{0,7,00,07,14,21,28,\ldots,91,98,000,007,014,\ldots,098,105,112,\ldots\}.$$

Recall that if $n \in \mathbb{Z}$, the expression $n \bmod 7$ refers to the unique integer, $a \in \{0,1,\ldots,6\}$ such that $n = 7p + a$ for some $p \in \mathbb{Z}$, and that if n, n' are integers, then $n - n'$ is divisible by 7 iff $n \bmod 7 = n' \bmod 7$.

(a) Show that for any integers $m, n \in \mathbb{Z}$, we have

$$(10m + n) \bmod 7 = \Bigl(3(m \bmod 7) + (n \bmod 7)\Bigr) \bmod 7.$$

Research supported in part by an NSERC grant.

(b) Use the previous part to design an 8-state DFA, M, that recognizes L. [Hint: it is easiest to name states in some convenient way so that you can describe the values of $\delta(q,\sigma)$ by a simple formula. You probably don't want to draw a graph that would need to depict $8 \cdot 10 = 80$ transition arrows...]

(c) Say that $L' = L \cup \{\epsilon\}$, so that L' is the language of strings representing integers divisible by 7, where we allow leading 0's and we do allow consider the empty string ϵ, to be part of L'.

(i) Describe a simple modification of M that yields another 8-state DFA, M, that recognizes L'.

(ii) Can you describe a 7-state DFA that recognizes L' ?

(4) Let L be the set of strings of bits, i.e., strings over the alphabet $\Sigma_{\rm bits} = \{0,1\}$, that represent integers in binary notation that are divisible by 3, where we allow leading 0's but we don't consider the empty string, ϵ, to be part of L. Hence the first few elements of L are

$$L = \{0, 00, 11, 000, 011, 110, 0000, 0011, 0110, 1001, 1100, 1111, \ldots\}.$$

(a) Give a 4-state DFA that recognizes L.

(b) Let $L' = L \cup \{\epsilon\}$. Give a 3-state DFA that recognizes L'.

(c) Let L'' the language L except that we do not allow leading 0's. Hence

$$L'' = \{0, 11, 110, 1001, 1100, 1111, \ldots\}$$

Give a 6-state DFA that recognizes L''.

(5) Let L' be as in the previous exercise, i.e.,

$$L' = L \cup \{\epsilon\} = \{\epsilon, 0, 00, 11, 000, 011, 110, 0000, 0011, 0110, 1001, 1100, 1111, \ldots\}.$$

Show that any DFA that recognizes L' must have at least three states, i.e., cannot have two states or fewer. Argue by contradiction: assume that a DFA with at most two states recognizing L' exists, and determine what the DFA must look like, and then obtain a contradiction. **Do not use the Pumping Lemma or Myhill-Nerode Theorem, which occur later in the textbook and in the course, but argue "from scratch."** [Hint: first explain why any DFA recognizing L' has at least one accepting state and one rejecting (i.e., not accepting) state. If there is only one accepcting state and one rejecting, which of these states must be the initial state? Next, since $0 \in L'$ and $1 \notin L'$, what are transitions out of the initial state (i.e., the values of $\delta(q,0)$ and $\delta(q,1)$ where q is the intial state)? Continue this line of reasoning.]

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA.
E-mail address: `jf@cs.ubc.ca`
URL: `http://www.cs.ubc.ca/~jf`