Marks

[10] **1.** Give a formal description of a Turing machine—and explain how your machine works—that takes as input, $x \in \{0,1\}^*$, and (1) accepts x if it has an even number of 1's, and (2) rejects x if not. You should **explicitly write** your choice of $Q, \Sigma, \Gamma, \delta, q_0, q_{\mathrm{accept}}, q_{\mathrm{reject}}$. For example, you should write $\Sigma = \{0,1\}$, since this is the input alphabet.

Continued on page 3

[16] **2.** (4 points for each part) Briefly justify your answers:

(a) Let $\mathcal{P}$ and $\mathcal{I}$ be any countably infinite sets and let "Result" be a function described in Axiom 1. Are there languages, i.e., subsets of $\mathcal{I}$, that are not recognized by any element of $\mathcal{P}$?

(b) Russell's paradox involves considering "the set of all sets that do not contain themselves." What is the standard way of resolving this paradox?

(c) Assume that you have a set of axioms in a system of logic where you can construct a sentence, S, whose meaning is "S is not provable (from the axioms)." Assume that you have an interpretation of your sentences so that each sentence is either true or false, but not both. Show that at least one of the following holds: (1) Some sentence that is provable (from the axioms) is false, or (2) Some sentence is true but not provable (from the axioms).

(d) In class we showed that $|S| < |Power(S)|$ for any set S, where $Power(S)$ is the set of all subsets of S. Our proof involved considering any map $f\colon S \to Power(S)$, and forming the set

$$T = \{s \in S \mid s \notin f(s)\}.$$

How do we use this T and what assumptions do we make on f to show that $|S| < |Power(S)|$?

Continued on page 4

[10] **3.** Let NO-COUNT-3 be the language of descriptions, $\langle p, i \rangle$, consisting of a 421Simple program, p, and an input, i, such that p has a variable named COUNT, and when p is run on input i, the variable COUNT that never attains the value 3 (at any point in its computation). Is NO-COUNT-3 decidable (say, by a 421Simple program) or recognizable? Justify your answer.

Continued on page 5

Continued on page 6

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Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - October 2014

Computer Science 421/501

Closed book examination | Time: 50 minutes

Name ____________________ **Signature** ____________________

Student Number ____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		10
2		16
3		10
Total		36