CPSC Final Practice 2023

(1) (a) True: The Boolean variables in the proof of the Cook-Levin theorem can be used to build a circuit of size $O(t(n)^2)$ for any Turing machine that takes time $t(n)$ (assuming $t(n) \geq n$ for all n).

(b) False: Formulas are very special cases of circuits; having a circuit of size $O(t(n)^2)$ does not imply that you have an equivalent circuit of size $O(t(n)^2)$ (or any poly(n)).

(c) True; see a homework problem this year, which reduces SUBSET-SUM to PARTITION.

(d) True; if $P = NP$, then any $L \in P$ s.t. $L \neq \emptyset$ or Σ^* for some Σ is NP-complete.

(e) True; done in class; an alternate method was done on the homework, using

$$\mathrm{Th}_{2,n}(x_1,\dots,x_n)$$

$$= \mathrm{Th}_{2,n/2}(x_1,\dots,x_{n/2}) \vee \mathrm{Th}_{2,n/2}(x_{n/2+1},\dots,x_n)$$

$$\vee \Big((x_1 \vee \dots \vee x_{n/2}) \wedge (x_{n/2+1} \vee \dots \vee x_n) \Big)$$

(f) True; a formula of size $O(n \log_2 n)$ in $x_1,\dots,x_n$ gives a circuit of size $O(n \log_2 n)$

(g) False; Subbotovskaya's 1961 result shows Parity_n requires formula size of $\geq n^{3/2}$ (later results proved $\geq n^{2-\varepsilon}$ for all $\varepsilon > 0$).

(h) True:

$$\mathrm{Parity}_n(x_1,\dots,x_n) = \mathrm{Parity}_{n/2}(x_1,\dots,x_{n/2})$$

$$\oplus\ \mathrm{Parity}_{n/2}(x_{n/2+1},\dots,x_n)$$

with n even shows that $L_n \leq 4 L_{n/2}$ for n even, where L_m = min size for Parity_m

(2) $x_1 \iff x_1 \vee x_1 \vee x_1$,

$x_2 \vee x_3 \vee x_4 \vee \neg x_5 \iff$

$(x_2 \vee x_3 \vee z_1) \wedge (\neg z_1 \vee x_4 \vee \neg x_5)$ is satisfiable,

hence both are true iff

$$(x_1 \vee x_1 \vee x_1) \wedge (x_2 \vee x_3 \vee z_1) \wedge (\neg z_1 \vee x_4 \vee \neg x_5)$$

is satisfiable.

(3) No — a 3CNF is either always true, or false on at least $\frac{1}{8}$ of the possible settings of its variables to T, F.

Since $x_2 \vee x_3 \vee x_4 \vee \neg x_5$ is false on only $\frac{1}{16}$ of the settings of its variables to T, F

(namely $x_2 = x_3 = x_4 = F$ and $x_5 = T$ and the other settings of its other variables),

$x_2 \vee x_3 \vee x_4 \vee \neg x_5$ cannot be written as a

3CNF.