**THE UNIVERSITY OF BRITISH COLUMBIA**
**CPSC 421/501: FINAL EXAMINATION - PART 2 - December 22, 2021**

Full Name: xLASTNAMEx CS Ugrad ID: xFIRSTNAMEx

Signature: ______________________ UBC Student #: [illegible]

**Important notes about this examination**

1. You have 60 minutes to complete this part of the exam.
2. The exam is closed book, and students may bring in two double-sided sheets of notes (8.5" x 11").
3. Good luck!

## Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
   i. speaking or communicating with other examination candidates, unless otherwise authorized;
   ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
   iii. purposely viewing the written papers of other examination candidates;
   iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
   v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).



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## 0. Identification

Please make sure that the following is your 5-character ugrad email id:

# `xFIRSTNAMEx`

**Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.**

1. (14 POINTS, 2 POINTS PER CORRECT T/F ANSWER — NO PENALTY FOR INCORRECT RESPONSES)

Circle either T for true, or F for false, for each of the statements below.

There are at most countably many Turing machines. T F

Turing machines $(Q, \Sigma, \Gamma, \ldots)$ can have $Q$ being an arbitrary set.

There are at most countably many standardized Turing machines. T F

Such Turing machines can be described as strings over $\{0,1,\ldots,9,\#\}$

For any alphabet, $\Sigma$, there are at most countably many languages over $\Sigma$ that are decidable by Turing machines. T F

By mapping $\Sigma$ to $\{1,\ldots,n_{|\Sigma|}\}$, such languages can be decided by standardized Turing machines

Let $L$ be an infinite language over $\Sigma = \{a\}$ such that $a^{29}, a^{40} \in L$ and $a^{41}, \ldots, a^{50} \notin L$. Then any DFA that recognizes $L$ must have at least 22 states. T F

$L = \{a^n \mid n \bmod 11 = 7\}$ requires only 11 states

Let $L$ be an infinite language over $\Sigma = \{a\}$ such that $a^{34}, a^{40} \in L$ and $a^{41}, \ldots, a^{50} \notin L$. Then any DFA that recognizes $L$ must have at least 17 states. T F

See homework this year. Using terminology from 2023, if $n_0 \leq 34$, then $p \geq 11$ since $a^{40} \in L$ and $a^{41}, \ldots, a^{50} \notin L$, and then

The acceptance problem for Turing machines is recognizable by a Turing machine. T F

$A_{TM}$ is recognized using a Universal T.M.

The acceptance problem for Turing machines is decidable by a Turing machine. T F

$A_{TM}$ is undecidable

$a^{34} \in L$ and $a^{41}, \ldots, a^{50} \notin L \Rightarrow p \geq 17$. Here

$n_0$ = smallest integer s.t. $\forall n \geq n_0$, $x^n \in L \Leftrightarrow x^{n+p} \in L$

where $p$ is the period of $L$. We know a DFA for $L$ requires $n_0 + p$ states.

2. (5 PARTS, 4 POINTS FOR EACH PART)

(a) Let $M$ be a fixed Turing machine. Is the set of all possible configurations of $M$ countably infinite? Explain.

Yes: A configuration consists ~~of~~ an element of $Q \times \mathbb{N}$ (tape head location) plus a string over $\Gamma$ for the tape symbols $\Gamma$. Since $Q, \mathbb{N}, \Gamma^*$ are countable so is $Q \times \mathbb{N} \times \Gamma^*$

(b) Is 3COLOUR in NP, i.e., can 3COLOUR (the descriptions of legally 3-colourable graphs) known to be verifiable in polynomial time? Explain.

Yes! one can non-deterministically guess the colouring, and check (or verify) that the colouring is legal.

(c) Let $L = \{w \in \{a,b\}^* \mid |w| \text{ is a perfect square}\}$, i.e., the words over $\{a,b\}$ whose length is $0, 1, 4, 9, 16, \ldots$. Is $L$ regular? Explain.

No: Since the smallest square $> n^2$ is $(n+1)^2$ (for any $n \in \mathbb{N}$), the shortest word in $\mathrm{AccFut}_L(a^{n^2+1})$ is of length $(n+1)^2 - n^2 - 1 = 2n$.

Hence as $n$ varies over $\mathbb{N}$, the values of $\mathrm{AccFut}_L(a^{n^2+1})$ are all different.

(d) Consider any Program-Input input system $(\mathcal{P}, \mathcal{I}, \text{Result})$ such that $\mathcal{P} = \mathcal{I} = \Sigma^*$ for some alphabet $\Sigma$. Is there a language that is not recognized by any program in $\mathcal{P}$? Explain.

Yes; $P = \Sigma^*$, the set of languages refers to $\text{Power}(\mathcal{I}) = \text{Power}(\Sigma^*)$, and by Cantor's Theorem there is no surjection $\Sigma^* \to \text{Power}(\Sigma^*)$.
Hence some language is not recognized by any element of $\mathcal{P}$.

(e) Let ACCEPTANCE be the acceptance problem over Turing machines and HALT be the halting problem over Turing machines. If $L$ is decidable by a Turing machine with oracle calls to ACCEPTANCE, is it also decidable by a Turing machine with oracle calls to HALT? Explain.

Yes. Given any oracle call of $\langle M, w \rangle$ to ACCEPTANCE, we can modify $\langle M \rangle$ to produce $\langle M' \rangle$ s.t. when $M$ accepts, then $M'$ accepts,
" " rejects, " " loops,
and " " loops, " " loops

(we simply replace $q_{rej}$ in $M$ by an infinite looping state to get $M'$).
Calling $\langle M', w \rangle$ to a HALT oracle returns the same answer as a $\langle M, w \rangle$ call to ACCEPTANCE.
Hence each oracle call to ACCEPTANCE can be replaced by an oracle call to HALT.