

THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: FINAL EXAMINATION - PART 1 - December 22, 2021

Full Name: xLASTNAMEx

CS Ugrad ID: xFIRSTNAMEx

Signature: _____

UBC Student #: XXXXXXXXXXXXX

Important notes about this examination

1. You have 60 minutes to complete this part of the exam.
2. The exam is closed book, and students may bring in two double-sided sheets of notes (8.5" x 11").
3. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).



0. IDENTIFICATION

Please make sure that the following is your 5-character ugrad email id:

xFIRSTNAMEx

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. (10 POINTS, 2 POINTS PER CORRECT T/F ANSWER — NO PENALTY FOR INCORRECT RESPONSES)

Circle either T for true, or F for false, for each of the statements below.

Let L be any language over $\{a, b\}$ that is recognized by some NFA with 30 states. Then there must exist a DFA that recognizes L with at most 900 states. T F

The language $\{a^{(3n+7)} \mid n \in \mathbb{N}\} = \{a^{10}, a^{13}, a^{16}, a^{19}, \dots\}$ is regular. T F

The language $\{a^{(n^2)} \mid n \in \mathbb{N}\} = \{a, a^4, a^9, a^{16}, \dots\}$ is regular. T F

The set of all regular languages over $\{a, b\}$ is countably infinite. T F

There exists a surjection from \mathbb{N} to $\{a, b\}^*$. T F

2. (15 POINTS)

Let $L = \{a, b\}^*\{aa, bb\}$, i.e., the language of words over $\Sigma = \{a, b\}$ that end in either aa or bb .

- (1) Write a formal description of a DFA (either by a diagram, table of δ values, or listing every value of a DFA) that recognizes L .
- (2) Use the Myhill-Nerode theorem (and possibly your solution to the previous part) to determine the minimum number of states of a DFA needed to recognize L (over the alphabet $\Sigma = \{a, b\}$).

3. (10 POINTS)

Let $L = \{a, b\}^* \{ab, ba\}$, i.e., the language of words over $\Sigma = \{a, b\}$ that end in either ab or ba . Write a formal description of a Turing machine that recognizes L , and give a **brief explanation (one to three sentences)** of how your Turing machine works; make sure you clearly indicate $Q, \delta, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}$ and **all values** of δ .

4. (10 POINTS)

Let

$$L = \{a^n b^{(n^2)} \mid n \in \mathbb{N}\} = \{ab, a^2b^4, a^3b^9, \dots\}.$$

Prove that L is not regular. [You may use the Myhill-Nerode theorem, or any other facts covered in our course.]