

**The University of British Columbia**  
Final Exam, December 5, 2017  
**CPSC 421**

Closed book examination

Time: 2.5 hours

Last Name \_\_\_\_\_ First \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_

**Special Instructions:**

Two two-sided 8.5 x 11 sheets of notes allowed.

**Student Conduct during Examinations**

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  - (a) speaking or communicating with other candidates, unless otherwise authorized;
  - (b) purposely exposing written papers to the view of other candidates or imaging devices;
  - (c) purposely viewing the written papers of other candidates;
  - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		16
2		15
3		15
4		15
5		15
Total		76

[16] 1. Circle either T (true) or F (false).

Circle either T for true, or F for false, for each of the statements below:

The set of functions  $\{0, 1\} \rightarrow \mathbb{Z}$  is countable. T F

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If  $L' \leq_P L$  then  $L' \in P^L$ . T F

If  $L$  is undecidable and recognizable, then  $L$ 's complement is unrecognizable. T F

If  $L$  and  $L'$  are in PSPACE then  $L^* \cap L'$  is also in PSPACE. T F

If  $L$  is PSPACE-complete and  $L' \in \text{NP}$ , then  $L' \leq_P L$ . T F

If  $L \in \text{P}$ , then there are polynomial size circuits for  $L$ . T F

[15] **2.** Give a Turing machine that takes as input,  $x \in \{0,1\}^*$ , and (1) accepts  $x$  if  $x$  begins with a 0 and  $x$  has exactly two more 0's than it has 1's, and (2) rejects  $x$  otherwise. You must **explain how your machine works**, and **explicitly write** your choice of  $Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}$ . To describe  $\delta$ , you may (1) list its values, or (2) use a diagram as used in Sipser's textbook (and class), or (3) give a table of its values as done in class and the solutions to Homework 10.

[15] **3.** Let  $L$  be the language of descriptions of a sequence of positive integers  $n_1, \dots, n_k, m, t$  such that there is an  $I \subset [k] = \{1, \dots, k\}$  and a  $J \subset [k]$  for which

$$\left( \sum_{i \in I} n_i \right) + m \left( \sum_{j \in J} n_j \right) = t.$$

Show that  $L$  is NP-complete; you may use the fact that SUBSET-SUM and PARTITION are known to be NP-complete.

[15] 4. Fix an integer  $k \in \mathbb{N}$ . Let  $L$  be the language of strings over  $0, 1$  whose  $k$ -th last symbol is a 1 (and whose length is therefore at least  $k$ ), i.e.,

$$L = \left\{ x1y \mid x, y \in \{0, 1\}^* \text{ and } |y| = k - 1 \right\}.$$

(a) Write an NFA that recognizes  $L$  and has at most  $k + 1$  states, and explain how your NFA works.

(b) Prove that any DFA recognizing  $L$  has at least  $2^k$  states.

[18] **5. Short Problems.** Each question is worth 3 points. Answer each question and **justify your answer. No credit will be given for a simple yes or no.**

(a) Let  $B$  be any PSPACE-complete language. Is  $P^B \subset \text{PSPACE}$ ?

(b) Is NP contained in  $\text{P}^{3\text{SAT}}$ ?

(c) Let  $L$  be the language of descriptions  $\langle M, w, q \rangle$  of a (deterministic) Turing machine  $M$ , an input  $w$  to  $M$ , and a state  $q$  of the Turing machine such that  $q$  is reached during the computation of  $M$  on input  $w$ . Is  $L$  recognizable?

- (d) Let SNEAKY-PSPACE be the descriptions  $\langle M, w, 1^s \rangle$  where  $M$  accepts  $w$  within space  $1^s$ . Prove that if  $L \in \text{PSPACE}$ , then  $L$  has a polynomial time reduction to SNEAKY-PSPACE.

- (e) If  $L$  is in P and  $L'$  is any language, then is  $\{s \mid st \in L \text{ for some } t \in L'\}$  necessarily in P? [Hint: Consider  $\{0^n 1^n\}$ .]