

Marks

- [10] 1. Give a formal description of a Turing machine—and explain how your machine works—that recognizes the language

$$L = \{0^n 10^n \mid n \text{ is a non-negative integer}\}.$$

You should **explicitly write** your choice of  $Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}}$ . You may use two tapes if you like. You may either write out the values of  $\delta$ , or depict these values in a state diagram.

[10] **2.**

(a) Give a DFA that recognizes the language

$$0^*1^* = \{0^n1^m \mid n, m \text{ are non-negative integers}\}.$$

You may either write out the values of  $\delta$ , or depict these values in a state diagram.

(b) Let  $L = \{0^n10^n \mid n \text{ is a non-negative integer}\}$ . What is  $\text{Future}(L, 0001)$ ? Explain.

(c) Show that the language  $L$  in part (b) cannot be recognized by any DFA.

- [10] **3.** In class and the textbook we showed that 3SAT is reducible to SUBSET-SUM by a polynomial time reduction,  $f$ . Briefly describe this reduction and justify that  $w \in 3\text{SAT}$  iff  $f(w) \in \text{SUBSET-SUM}$ . Explain why  $f$  takes roughly quadratic time to compute. [Recall that 3SAT is the set of 3CNF formulas that are satisfiable, and SUBSET-SUM is the set of integers  $n_1, n_2, \dots, n_r, t$  such that for some  $I \subset \{1, \dots, r\}$  we have  $\sum_{i \in I} n_i = t$ .]

- [10] 4. Let DOUBLE-SAT be the set of descriptions of Boolean formulas that have at least two satisfying assignments. Show that DOUBLE-SAT is NP-complete. [Hint: You may use the fact that SAT is NP-complete. Don't forget to show that DOUBLE-SAT is in NP.]

[32] 5. 4 points per part. **Briefly justify** your answers; you will not get credit for just writing “yes” or “no” (or any short answer without justification).

(a) Show that if  $C_1, C_2, \dots$  are countable sets, then  $C_1 \cup C_2 \cup \dots$  is countable.

(b) What does Savitch’s theorem assert, and why does this show that NPSPACE (non-deterministic polynomial space) is equal to PSPACE (polynomial space)?

(c) Let  $A$  be any problem that is complete for PSPACE under polynomial time reductions. Is PSPACE contained in  $P^A$ ?

(d) Let  $L_{NP \text{ easy}}$  consist of all descriptions of a triple,  $\langle M, i, t \rangle$  where  $M$  is a non-deterministic Turing machine that accepts input  $i$ , running in time  $t$  where  $t$  is expressed in unary. Show that any language in  $NP$  can be reduced to  $L_{NP \text{ easy}}$  by a polynomial time reduction.

- (e) Let HALT be the Halting Problem for Turing machines, and let  $\text{HALT}^{\text{HALT}}$  be the Halting Problem for oracle Turing machines with oracle HALT. Is there an oracle Turing machine with oracle HALT that can recognize  $\text{HALT}^{\text{HALT}}$ ?
- (f) Same problem as the last part, except with “recognize” replaced with “decide.”
- (g) Say that you are given a description,  $\langle A, w \rangle$ , of an NFA (non-deterministic finite automaton),  $A = (Q, \Sigma, \delta, q_0, F)$  and a word,  $w \in \Sigma^*$ , and that the transition function  $\delta: Q \times \Sigma \rightarrow \text{Power}(Q)$  is written out by each of its values. Can you decide if  $A$  accepts  $w$  in polynomial time of the length of  $\langle A, w \rangle$ ?
- (h) For each non-negative integer,  $n$ , let  $\text{SAT}_n$  be those strings of length  $n$  that lie in SAT. If SAT is in P, how do we prove that  $\text{SAT}_n$  has polynomial size circuits? How does this related to the question of P versus NP?

**The End**

Be sure that this examination has 8 pages including this cover

The University of British Columbia

Final Examinations - December 2014

Computer Science 421/501

Closed book examination

Time: 150 minutes

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

**Special Instructions:**

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

**Rules governing examinations**

**1. Each candidate should be prepared to produce his library/AMS card upon request.**

**2. Read and observe the following rules:**

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

**3. Smoking is not permitted during examinations.**

1		10
2		10
3		10
4		10
5		32
Total		72