Marks

[8] **1.** Give an explicit description of a Turing machine that takes as input, $x \in \{0,1\}^*$, and (1) accepts x if the first character of x equals the last character, and (2) rejects x if not. You should **explicitly write** your choice of $Q, \Sigma, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$ and **intuitively explain** how the machine works. For example, you should write $\Sigma = \{0,1\}$, since this is the input alphabet.

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[8] **2.** Let Σ be a finite, nonempty alphabet.

(a) Show that Σ^* is infinite but countable.

(b) Is the set of subsets of Σ^* countable? Justify your answer.

(c) Explain the relevance of parts (a) and (b) to computability in the situation where you have a set of programs that are strings over Σ, all of which take their inputs from strings over Σ.

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[8] **3.** Outline how to reduce 3SAT to SUBSET-SUM; illustrate this reduction on a simple example.

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[8] **4.** Show that if $L_1 \leq_P L_2$, i.e., L_1 is polynomial time reducible to L_2, and if $L_2 \leq_P L_3$, then $L_1 \leq_P L_3$. If the $L_1 \leq_P L_2$ reduction takes time order n^5, and the $L_2 \leq_P L_3$ takes time order n^9, give a bound on the time the $L_1 \leq_P L_3$ will require.

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[8] **5.** Let DOUBLE-SAT be the language of Boolean formulas that have at least two satisfying assignments. Show that DOUBLE-SAT is NP-complete.

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[8] **6.** Recall how we showed L_{yes} is undecidable. Assume to the contrary that there is a program, P, that decides L_{yes}. Let D be a program such that for all programs, Q,

$$\text{Result}(D, \text{EncodeProg}(Q))$$

$$= \neg\text{Result}(P, \text{EncodeBoth}(Q, \text{EncodeProg}(Q)))$$

Argue that considering the value of $\text{Result}(D, \text{EncodeProg}(D))$ leads to a contradition.

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[8] **7.** In this course we studied problems that cannot be solved (via counting arguments or self-referential constructions) and that seem hard to solve quickly (NP-complete problems). In two or three paragraphs, outline the techniques for proving these results, and describe problems that you might encounter in practice that would relate to these results.

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[8] **8.** Use the Myhill-Nerode theorem to show that:

(a) $L = \{x \in \{0,1\}^* \mid x \text{ contains 01 as a substring}\}$ is regular (i.e., recognized by a DFA), and

(b) $L = \{x \in \{0,1\}^* \mid x = 0^n 1^n \text{ for some } n \geq 0\}$ is not regular.

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The University of British Columbia

Final Examinations - December 2011

Mathematics 421/501–101

Closed book examination | Time: 2.5 hours

Name ________________ **Signature** ________________

Student Number________________ **Instructor's Name** ________________

Section Number ________________

Special Instructions:

THIS EXAM IS TWO-SIDED! Calculators, other notes, or other aids may not be used. Answer the questions on the exam.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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