

2004 Brief Solutions to Final, Selected Problems

Note Title

12/1/2007

I(a) Yes - $L_1 \cap L_2 = \overline{(L_1 \cup \overline{L_2})}$, and reg langs closed under \cup (from reg expressions), and complementation (by exchanging accepting and nonaccepting states of DFA).

I(b) No - let L_1 be non-regular (e.g. $\{0^n 1^n\}$) and $L_2 = \overline{L_1}$. Then $L_1 \cup L_2 = \Sigma^*$ is regular, while L_1, L_2 are not.

Sorry

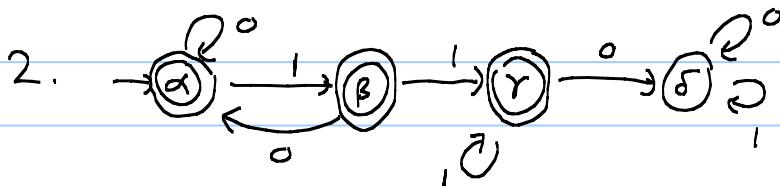
for
the
repeated

(b)'s I(b) Yes, just run the TMs for L_1 and L_2 simultaneously.

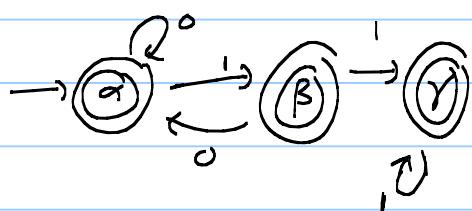
I(b) No - A_{TM} is acceptable, but not $\overline{A_{TM}}$.

I(c) Yes - just switch accepting and rejecting states.

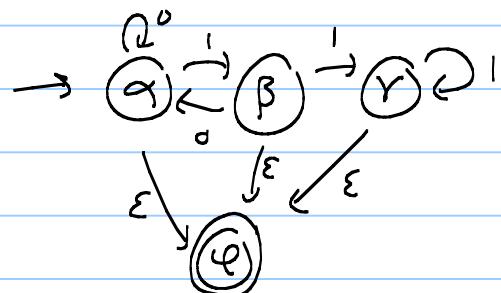
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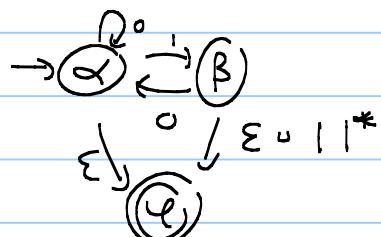
GNFA's: remove δ :



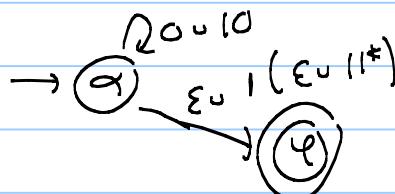
add unique final state



remove γ :



remove β :



reg exp: $(0 \cup 1)^*(\epsilon \cup 1(\epsilon \cup 1^*))$.

3. See text — pump $0^p 1^p 0^p 1^p$

5. See class notes & text.

6. Construct a TM that runs in stages 1, 2, 3, ... such that on stage i we simulate M for i steps on all strings of length $\leq i$, and halts whenever M accepts 5 strings at any stage. This TM accepts 5PLUS.

Given $\langle M, w \rangle$, let $R = \text{function of } M, w$ be the TM that ignores its input and simulates M on w (and accepts when M accepts).

Then

$$\langle M, w \rangle \in A_{\text{TM}} \Leftrightarrow R = R(M, w) \in 5\text{PLUS}.$$

Hence $A_{\text{TM}} \leq_m 5\text{PLUS}$ so 5PLUS is not decidable.

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7. See class notes, text, and/or note sheet for final 2007.

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8. Idea: $3\text{SAT} \leq_p \text{DOUBLE-SAT}$ by adding an extra Boolean variable to the 3SAT instance, so the 3CNF is satisfiable iff DOUBLE-SAT has at least 2 satisfying assignments.

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9. See homework solutions.