Midterm Practice ~ Some Solutions Oct. 30, 2023

Contents:

Homework 7: Individual, Problem (3)

Let $L = C_1^{rev} = \{ w \in \Sigma^* \mid w \text{ begins with an } a \}$

$\Sigma = \{a,b\}$. Let

$S_1 = \mathrm{AccFut}_L(\varepsilon) = a\Sigma^*$ (here

$S_2 = \mathrm{AccFut}_L(a) = \Sigma^*$

$S_3 = \mathrm{AccFut}_L(b) = \emptyset$

Then: Since $S_3 = \emptyset$ and $S_1, S_2 \neq \emptyset$, we have $S_3 \neq S_1$ and $S_3 \neq S_2$.

Since $\varepsilon \in S_2$ but $\varepsilon \notin S_1$, $S_2 \neq S_1$.

Hence S_1, S_2, S_3 are distinct, and hence any

DFA accepting $L = C_1^{rev}$ requires at least 3 states

Midterm Practice 2021: Some solutions (ignoring problems 5, 6, 9)

(2021, 1) This is <u>very similar</u> to Group Homework 7, Problem (2), which does the same for C_3, instead of C_4.

(1a) $\mathrm{AccFut}_{C_4}(aaa) = \mathrm{AccFut}_{C_4}(baaa)$,

since both:

(i) don't contain ε ($aaa, baaa \notin C_4$);

(ii) both contain Σ, since for $\sigma_1 \in \Sigma$, both $aaa\sigma_1$, $baaa\sigma_1$ have their 4th to last symbol equal to a;

(iii) similarly, both contain Σ^2, Σ^3;

(iv) for any $s \in \Sigma^*$ of length 4 or more, and for any $w \in \Sigma^*$, $ws \in C_4$ iff $s \in C_4$

(since the 4^{th} last symbol of ws lies in S).

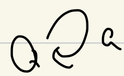
Hence

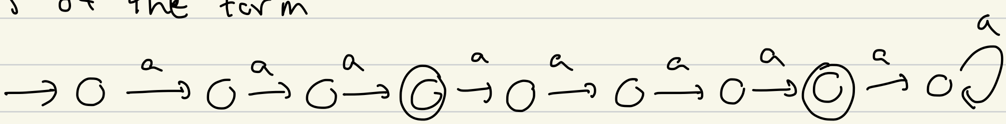
$$\mathrm{AccFut}_{C_4}(aaa) = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup C_4$$
$$= \mathrm{AccFut}_{C_4}(baaa)$$

(1b, 1c, 1d) Similar to Homework 7, Individual Problem 3.

(2021, 2) Similar to (2021, 1).

(2021, 3) Since $a^n \notin L$ if $n \geq 8$, we have $a^n \in L \iff a^{n+1} \in L$ for $n \geq 8$. Hence L has period 1. Hence the smallest DFA for L has cyclic part

Since $a^7 \in L$, the last state before  is accepting, and hence the smallest DFA is of the form



which has 9 states.

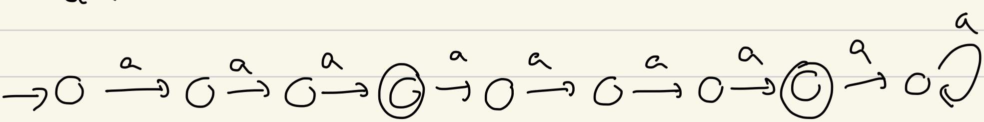
Alternative 1: Since L has period 1 and $a^n \notin L$ for $n \geq 8$, the fact that $a^7 \in L$ shows that

$$a^n \in L \Leftrightarrow a^{n+1} \in L$$

holds for $n \geq 8$, but not for $n \geq 7$. Hence,

So by Homework 5, (2e) [which is the same as (6.1.2(e)) in the handout], $n_0 = 8$ and $p = 1$ and the smallest DFA for L has $n_0 + p = 9$ states

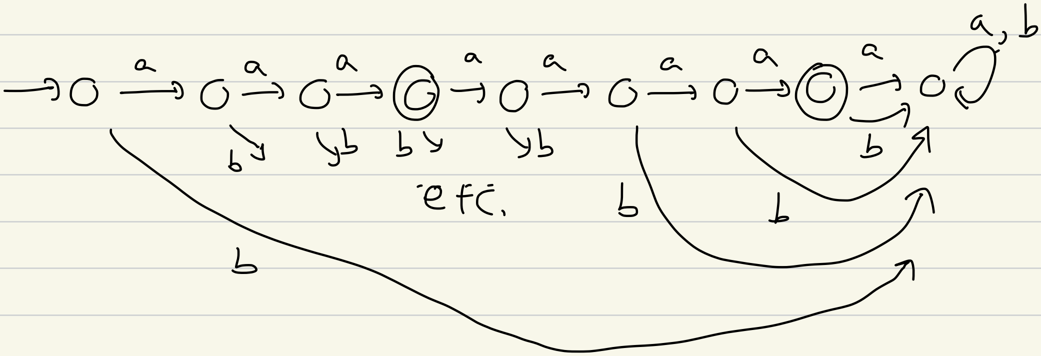
Alternative 2: Use Homework 5, Prob (1), i.e. 6.1.1, any DFA must have at least 9 states, and



accepts L and has 9 states

Alternative 3: Use Myhill-Nerode: for all n, consider the longest word in $\mathrm{AccFut}_L(a^n)$. Details omitted.

Second part: If $\Sigma = \{a, b\}$, you still need ≥ 9 states, since by erasing the arrows labelled "b" from any DFA for L over $\Sigma = \{a,b\}$, you get a DFA for L over $\Sigma = \{a\}$. But



recognizes L and has 9 states.

Alternative 1: Use Myhill-Nerode

(2021, 4) Similar to (3): L has period 4, and $n_0 = 3$, i.e. $a^n \in L \Leftrightarrow a^{4+n} \in L$ for all $n \geq 3$: $a^2 \notin L$ but $a^6 \in L$, so $n_0 \geq 3$.

But for $n \geq 3$, we have $a^n \in L$ iff $n \bmod 4 = 2$.

Hence $n_0 = 3$. So min # states is $n_0 + p = 7$.

Alt: Use Myhill-Nerode (likely longer)

(2021,7) In brief: you should be able to get a contradiction either way. Hence "John feeds those who don't feed themselves" gives a contradiction regarding John.

(2021,8) Batiste loves Batiste, hence Batiste $\notin S$. But if S' = set of people whom Batiste loves, then S' is everyone, hence Batiste $\in S'$. Since Batiste $\notin S$ and Batiste $\in S'$, $S \neq S'$. Hence the two sets of humans are different.

– Midterm Practice 2023: Some solutions

(2023, 1) Brief solutions, some explanations left out...

NON-ACCEPTANCE is unrecognizable, since otherwise we could use a recognizer for NON-ACCEPTANCE to recognize NON-SELF-ACCEPTANCE, which gives a contradiction (see (2023, 2) below or class notes).

NON-PYTHON is decidable (see class discussion)

ACCEPTANCE is recognizable (by a universal Python program), but undecidable, for otherwise

$$\Sigma^* \setminus (\text{ACCEPTANCE} \cup \text{NON-PYTHON})$$

$= \text{NON-ACCEPTANCE}$ would be decidable, In brief: one can reduce ACCEPTANCE to HALTING or REJECTING, so one can similarly argue that the latter two are undecidable (and both can be recognized using a universal TM).

NON-ACCEPTANCE can be reduced to LOOPING, hence the latter is unrecognizable.

(2023,2) We took $S = \Sigma_{ASCII}^*$,

$f = \text{LanguageRecBy}$, a map

$f : \Sigma_{ASCII}^* \to \text{Power}(\Sigma_{ASCII}^*)$.

Hence a language is recognizable iff it is in the image of f. By Cantor's theorem $T = \{ p \in \Sigma_{ASCII}^* \mid p \notin f(p) \}$ is not in the image, and hence T is not recognizable

2023: (3) – (5) <u>In brief</u> (you have to explain why)

(3a) True: $w \in L_1 \setminus L_2 \Leftrightarrow$

$w \in L_1$ AND $w \notin L_2$. We can run a decider for L_1, and then one for L_2 to test if $w \in L_1$ and if $w \notin L_2$ and accept or reject accordingly.

(3b) Let $L_1 = L_2$ be an undecidable language, e.g. ACCEPTANCE. Then $L_1 \setminus L_2 = \emptyset$, which is decidable.

(3d) similarly as (3b)

(3c) False: take $L_1 = \Sigma^*$,

$L_2 =$ NON-PYTHON $\cup$ ACCEPTANCE. Etc.

("Etc." means you have to explain this)

(4a): True. Etc.

(4b): False: e.g. take ACCEPTANCE and its complement. Etc.

(4c): True. Etc.

(4d): False: This seems more difficult. Let

$L_1 = \text{NON-ACCEPTANCE} \cup \{ \langle p, i \rangle \mid i \text{ begins with "a"} \}$

$L_2 = \text{NON-ACCEPTANCE} \cup \{ \langle p, i \rangle \mid i \text{ does not begin with "a"} \}$

Then both L_1, L_2 are unrecognizable, since NON-ACCEPTANCE can be reduced to L_1, L_2 (why?), but $L_1 \cup L_2 = \Sigma^*_{ASCII} \setminus \text{NON-PYTHON}$

(2023, 5) Etc.