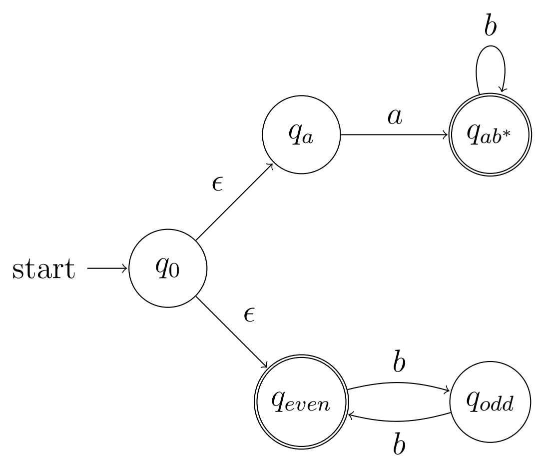
Midterm Solutions

CPSC 421/501, Fall 2020

Midterm 1

Problem (1) (5 Marks)



The upper branch decides ab^* and the lower branch decides $(b^2)^*$.

- q_0 initial state, allow us to take the union of the two branches using the epsilon jumps.
- q_a corresponds to when the letter seen is a.
- q_{ab^*} corresponds to when the letters seen are ab^*.
- q_{even} we've seen an continuous string of bs with even length (or we've seen no letters at all).
- q_{odd} we've seen an continuous string of bs with odd length.

Problem (2)

Part 1 (5 Marks): Note the following accepting futures of the language L:

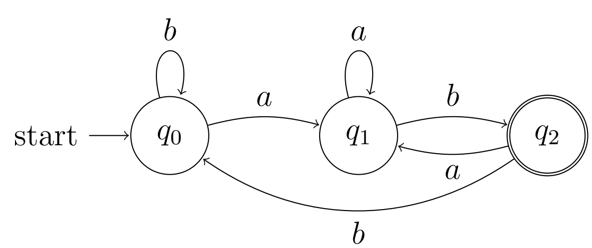
$\text{AccFut}_L(\epsilon) = L$
$\text{AccFut}_L(a) = L \cup \{b\}$
$\text{AccFut}_L(ab) = \{\epsilon\} \cup L$

We note that the language L has at least three distinct accepting futures and therefore by the Myhill-Nerode theorem any DFA that accepts L must have at least three states.

Not needed in the solutions but to show that three is the best lower bound you can say:

$\text{AccFut}_L((a,b)^*a) = \text{AccFut}_L(a)$
$\text{AccFut}_L((a,b)^*bb) = \text{AccFut}_L(\epsilon)$
$\text{AccFut}_L((a,b)^*ab) = \text{AccFut}_L(ab)$

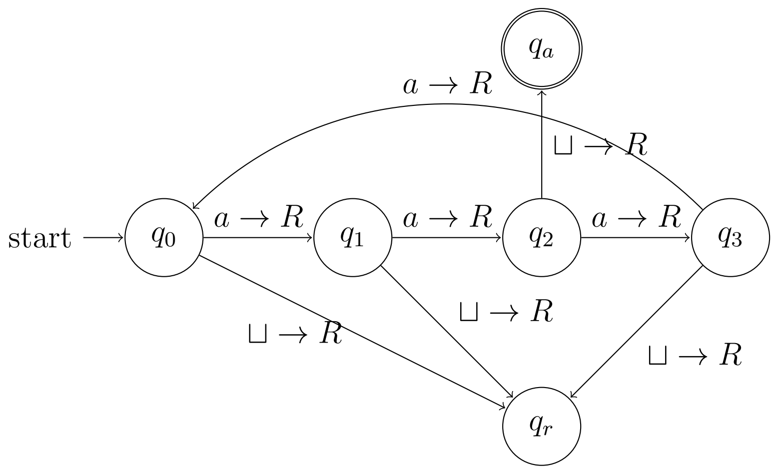
Part 2 (5 Marks): DFA with three states for the language L:



- q_0 corresponds to when the last letter seen is not a.
- q_1 corresponds to when the last letter seen is a.
- q_2 corresponds to when the last two letters seen are ab.

Problem (3) (10 Marks)

Our Turing machine has states $Q = \{q_0, q_1, q_2, q_3, q_a, q_r\}$ where q_0 is the initial state, q_a is the accepting state, and q_r is the rejecting state. The tape language is $\Gamma = \{a, \sqcup\}$. Then the Turing machine that recognizes L.



States q_0, q_1, q_2, and q_3 of the Turing machine correspond to the number of a's having been seen on the tape so far mod 4 being equivalent to $0, 1, 2,$ and 3 respectively. The Turing machine works by reading the string of a's on the tape until reaching the first blank cell and during this process it cycles through states $q_0, q_1, q_2,$ and q_3, keeping track of the value of the number of a's having been seen on the tape so far mod 4. When the Turing machine sees the first blank cell it accepts if and only if it is in state q_2, otherwise it rejects.

Midterm 2

Problem (1) (5 Marks)

Since $f(c) = \{a, c\}$, then $c \in f(c)$. For the definition of T we have $c \notin T$ precisely because $c \in f(c)$. Therefore, $f(c) \neq T$.

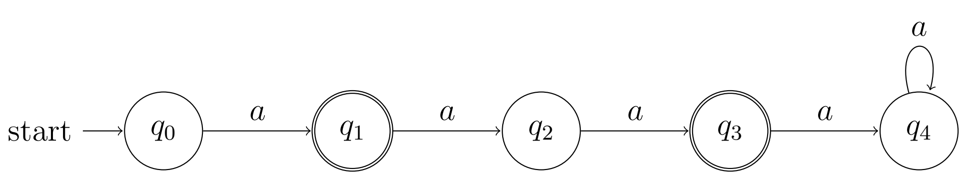
Problem (2)

Part 1 (5 Marks): Note the following accepting futures of the language L:

$\mathrm{AccFut}_L(\epsilon) = L$
$\mathrm{AccFut}_L(a) = \{\epsilon, a^2\}$
$\mathrm{AccFut}_L(a^2) = \{a\}$
$\mathrm{AccFut}_L(a^3) = \{\epsilon\}$
$\mathrm{AccFut}_L(a^k) = \varnothing$ for $k \geq 4$

We note that the language L has at five distinct accepting futures and therefore by the Myhill-Nerode theorem any DFA that accepts L must have at least five states.

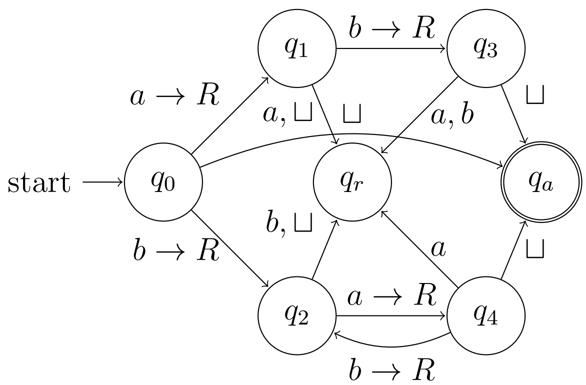
Part 2 (5 Marks): DFA with three states for the language L:



- q_0 corresponds to when we've seen ϵ.
- q_1 corresponds to when we've seen a.
- q_2 corresponds to when we've seen aa.
- q_3 corresponds to when we've seen aaa.
- q_4 corresponds to when we've seen a^k for $k \geq 4$.

Problem (3) (10 Marks)

Our Turing machine has states $Q = \{q_0, q_1, q_2, q_3, q_4, q_a, q_r\}$ where q_0 is the initial state, q_a is the accepting state, and q_r is the rejecting state. The tape language is $\Gamma = \{a, b, \sqcup\}$. Then the Turing machine that recognizes L.



Description left out since this question is similar to the first midterm.