CPSC 421/501 Nov 9, 2021

- Exams will be returned on gradescope likely next week

- Homework 7 will be assigned today and due Friday, Nov 19 (5 business days later)

- Don't forget your COVID test if travelling and is appropriate

- Homework 8 due Thursday, Nov 25

Pant! Today we build a universal TM...

(we'll need multi-tape machine)

Start:

$$MULT = \left\{ w_1 \# w_2 \# w_3 \;\middle|\; \begin{array}{l} w_1, w_2 \\ w_3 \in \{0,1\}^* \end{array} \text{ s.t. } \begin{array}{l} (w_1)_{\substack{\text{binary}\\\text{int}}} \cdot (w_2)_{\substack{\text{binary}\\\text{int}}} \\ = (w_3)_{\substack{\text{binary}\\\text{int}}} \end{array} \right\}$$

— high-level description

— med-level, implementation level

— formal description — give δ

Note: MULT is a lot harder to program than Example 3.11, 3.12

=

Ch 3:

single tape TM $\longrightarrow$ multi tape

$\delta: Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

$\delta:$ $Q \times \Gamma^k \to Q \times \Gamma^k \times \{L, R, S\}^k$

$\downarrow$ (from single tape TM) $\qquad$ $\downarrow$ (from multi tape)

non-det TM $\longrightarrow$ multi tape non-det

$\delta: Q \times \Gamma \to \text{POWER}(Q \times \Gamma \times \{L, R\})$

Really:

focus

single type → multi type

non-det

Ch 7

oracle TM

Ch 6
Ch 9

(multi-type
non-det
oracle call)

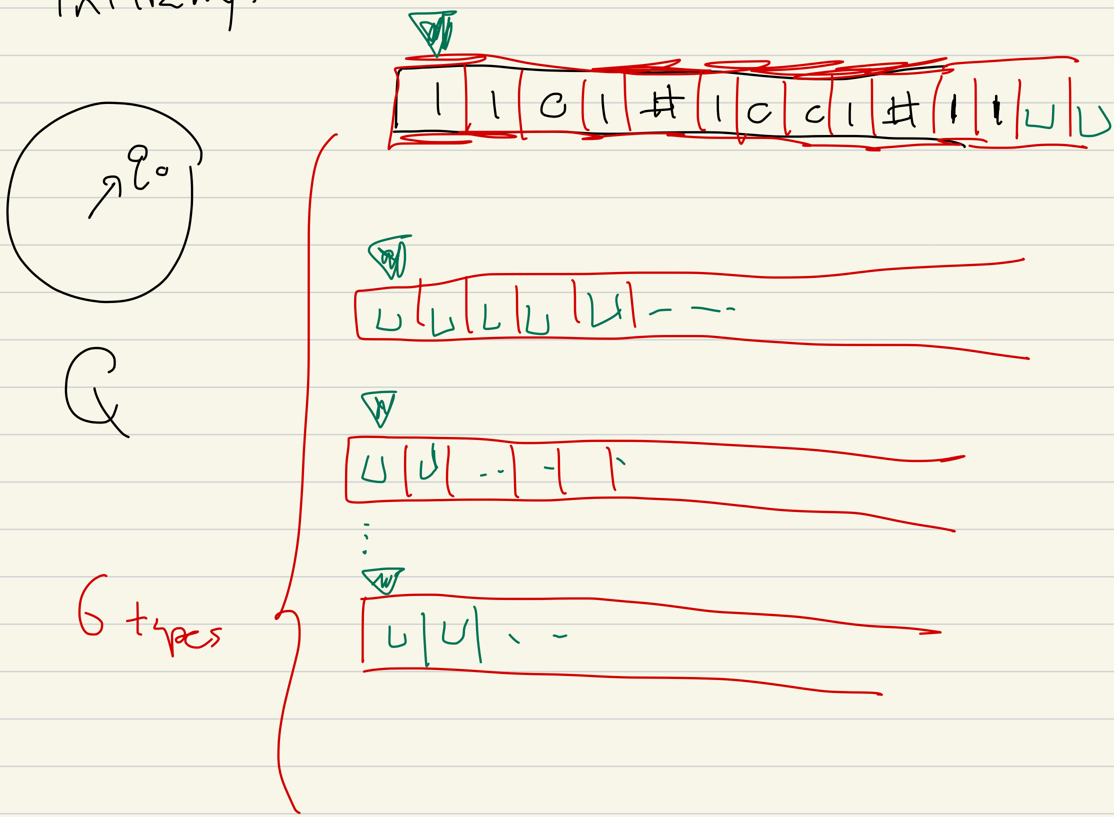
$$MULT = \{ w_1 \# w_2 \# w_3 \mid w_1, w_2, w_3 \in \{0,1\}^* \text{ s.t. } (w_1)_{\text{binary int}} \cdot (w_2)_{\text{binary int}} = (w_3)_{\text{binary int}} \}$$

$$\begin{array}{rrl}
 & 1101 & \leftarrow n \text{ bits} \\
\times & 1001 & \leftarrow m \text{ bits} \\
\hline
 & 1101 & \leftarrow \\
 & 0000 & \leftarrow \\
 & 0000 & \leftarrow \\
+ & 1101 & \leftarrow \\
\hline
 & 1110101 &
\end{array}$$

m things adding together

multi-tape:

initially:



$\Sigma = \{1, 0, \#\}$

$\Gamma = \{1, 0, \#\} \cup \{\sqcup\} \cup$ any finite set of additional symbols

Formally: Algorithm

$Q, \Sigma, \Gamma, q_0, q_{acc}, q_{rej}$

+

$\delta: Q \times \Gamma^6 \to Q \times \Gamma^6 \times \{L, R, S\}^6$

for 6-tape machine

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Say

$$\begin{array}{r} 1101 \\ \times\ 1001 \\ \hline \end{array}$$

check

v.s.

11

Since # tapes is fixed:

high-level description:

To understand the rules:

$\text{PALINDROME}_{\{a,b\}}$ takes

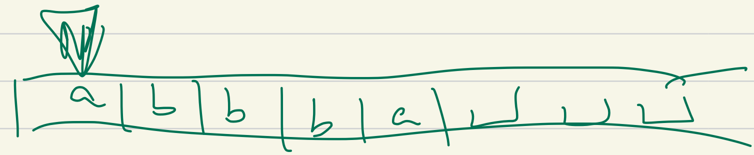
time at least order n^2

on a 1-tape machine

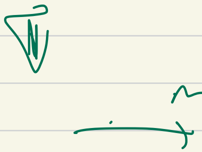
$$\left\{ w \in \{a,b\}^* \;\middle|\; w^{rev} = w \right\}$$

$w = \sigma_1 \ldots \sigma_n, \qquad \sigma_i \in \{a,b\}$

$w^{rev} = \sigma_n \sigma_{n-1} \cdots \sigma_1$



algorithm



move

end of tape, first ⊔

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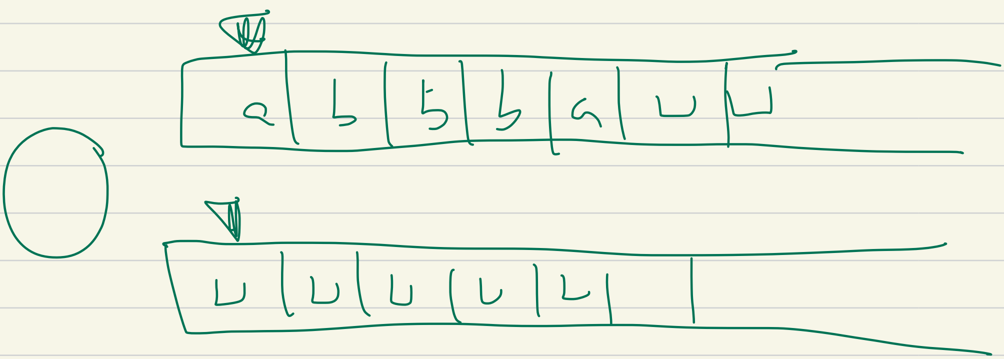
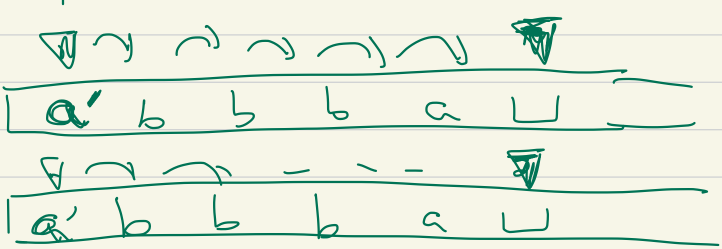
2 tape:



Alg: Copy input tape 1 onto tape 2

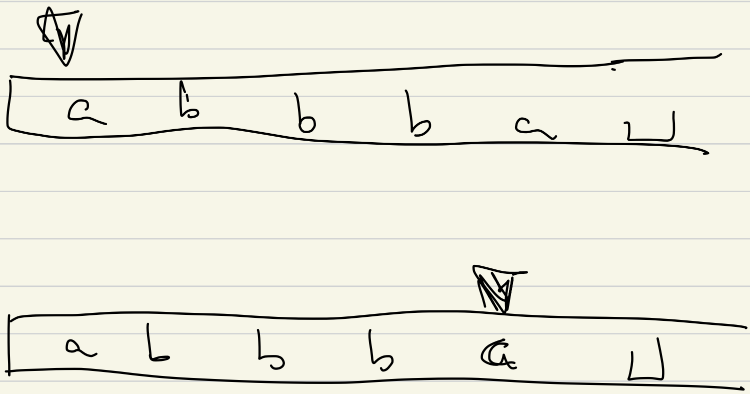
here



1

Then move one head start

2



then move type 1 head → R

type 2 head → L

compare

\# steps is linear.

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In phase 2 want S = stay option on the tape head

Break at 10:18 to 10:23

N.B.

PALINDROME requires

(1) order n^2 time on 1-tape

(no proof given in any of my CPSC 421/501 classes yet...)

(2) only order n time on 2-tape

[Sip]: $\delta(q_1, a) = (q_3, a, R)$ written

$$q_1 \xrightarrow{a \to R} q_3$$

High-level descript of

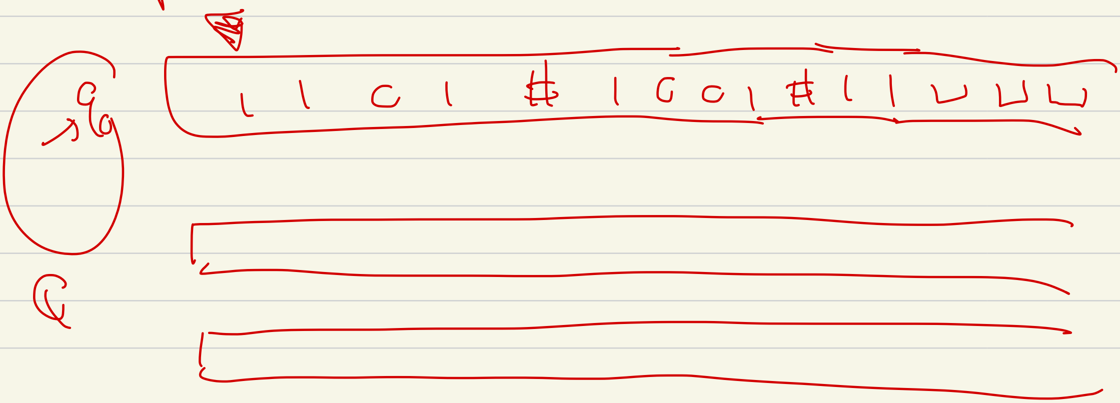
① MULT

High-level descript of

② Universal TM

① High level:

tape



Rem: For now, the model is

blah blah blah q_{acc} q_{rej} blah

needs to be a
decision problem

Soon (Ch 5 ...)

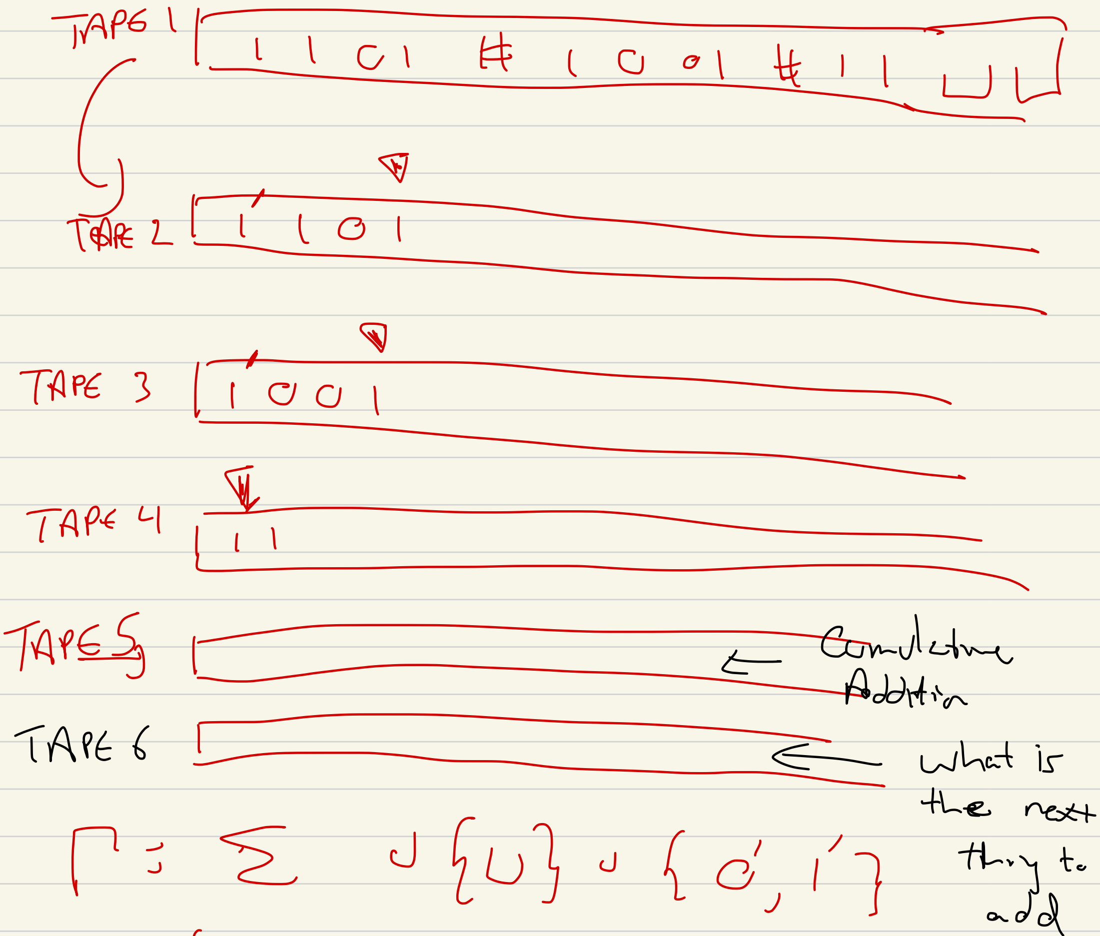
We actually have

tape1 | input

we'll print an "output" — tape1
— tape 2

Back to MULT: we could, at cost adding 3 tapes,

STEP



$$\Gamma = \underset{\{0,1,\#\}}{\Sigma} \cup \{\sqcup\} \cup \{0', 1'\}$$

Universal TM:

$$TM = (Q, \Sigma, \Gamma, q_0, q_{acc}, q_{rej}, \delta)$$

no harm, from the point of view of algorithms, of insisting

$$\boxed{Q = \{1, 2, \ldots, q\} \quad q \in \mathbb{N}}$$

insist

$$\left.\begin{array}{l} q_0 = 1 \\ q_{acc} = 2 \\ q_{rej} = 3 \end{array}\right\}$$ or write down these integers in $[q] = \{1, 2, \ldots, q\}$

$$\Sigma = \{1, 2, \dots, s\}$$

$s \in \mathbb{N}$

$$\Gamma = \{1, 2, \dots, s, s+1, \dots, t\}$$

$\uparrow$

$\sqcup$

$t \in \mathbb{N}$

Now $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$

TM $\Leftrightarrow$ really "standardized" TM

$\Leftrightarrow$ q, s, t, δ :

$\delta : \{1, \dots, q\} \times \{1, \dots, t\} \to \{1, \dots, q\} \times \{1, \dots, t\} \times \{L, R\}$

Class ends