CPSC 421/501 Nov 2, 2021

- Study guide for Midterm:
  Look at top of course webpage, News section, Oct 29 entry.

- Midterm will 60 minutes.
  You will be seated in some order, likely alphabetical by last name as it appears on the Faculty Service Centre

- Please remain outside the classroom until you are asked to enter the classroom

- You will asked to enter the classroom at roughly 9:35 am. The exam begins at 9:40 am.

- You should be able to do all the homework problems.

Today!

- Turing Machines & their variants

Goal!

(1) - Convince you that P = poly time on a 1-tape TM
(reasonable notion that agrees with poly time CPSC 320, poly time in C, Javascript, ...)

(2) Convince you that you can build a universal TM

① Multi-type machines:

seems "more realistic" and helpful in goals ① & ②

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Last time

$PALINDROME_{\{a,b\}}$

$= \{ w \in \{a,b\}^* \mid w = w^{rev} \}$

contains abba, aba, aabaa

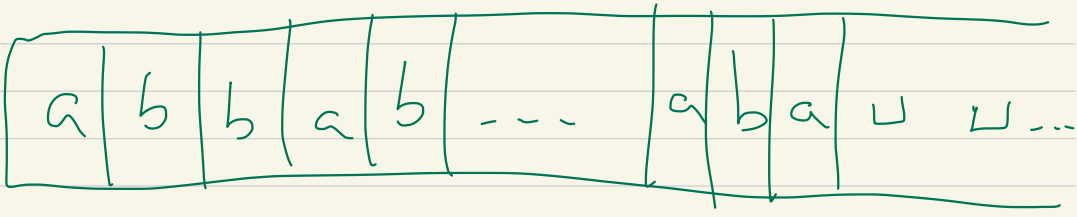
not to abbaa, ab, ba, --

Seemed excessively unrealistic to recognize

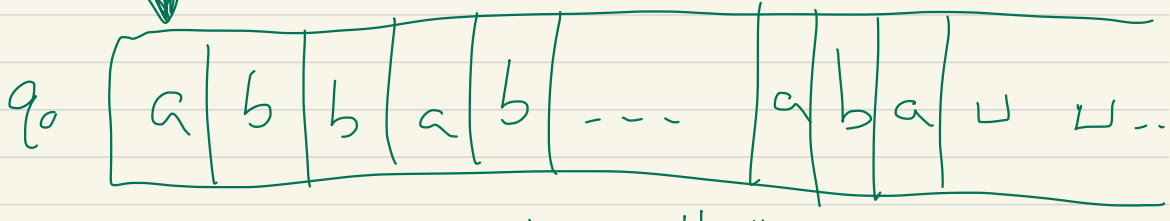
decide (decide = recognize + always halt)

on a 1-tape, classical TM

input

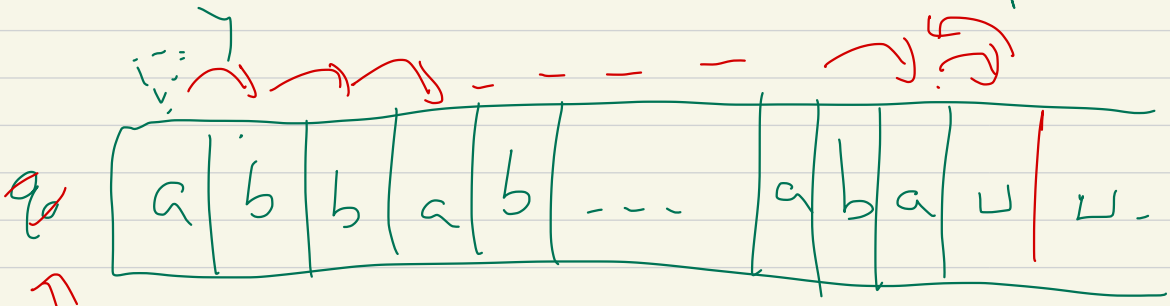


initially



Algorithm: remember cell # 1,

but we move to the end of tape



branch that remember cell 1 content

steps

$n + \log 2$

$n = |w|$ = size of input

then go back

Time of our algorithm:

$$c\left(n + (n-2) + (n-4) + \cdots\right)$$

$$= O(n^2)$$

maybe

$$\left(n + (n-1) + (n-2) \cdots\right) + O(1) \cdot n$$

maybe

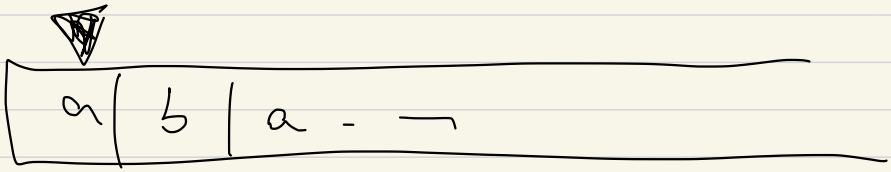
$$\frac{n(n+1)}{2} + O(n)$$

roughly

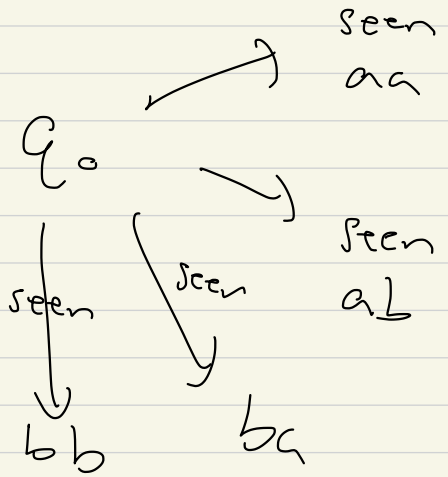
$$\frac{1}{2}n^2 + O(n)$$

could we improve this running time?

We can improve this:



read 1st & 2nd cell:



make "program," Q, twice as large, you can speed up by

to roughly $\frac{1}{4} n^2 + O(n)$

Really there is an algorithm that achieves, for any $c > 0$

$$c\, n^2 + \underbrace{O(n)}_{\text{maybe depends on } c}$$

Fixed Language = PALINDROME,

$\Sigma = \{a, b\}$

Q, Γ, state set Q

work tape alphabet Γ

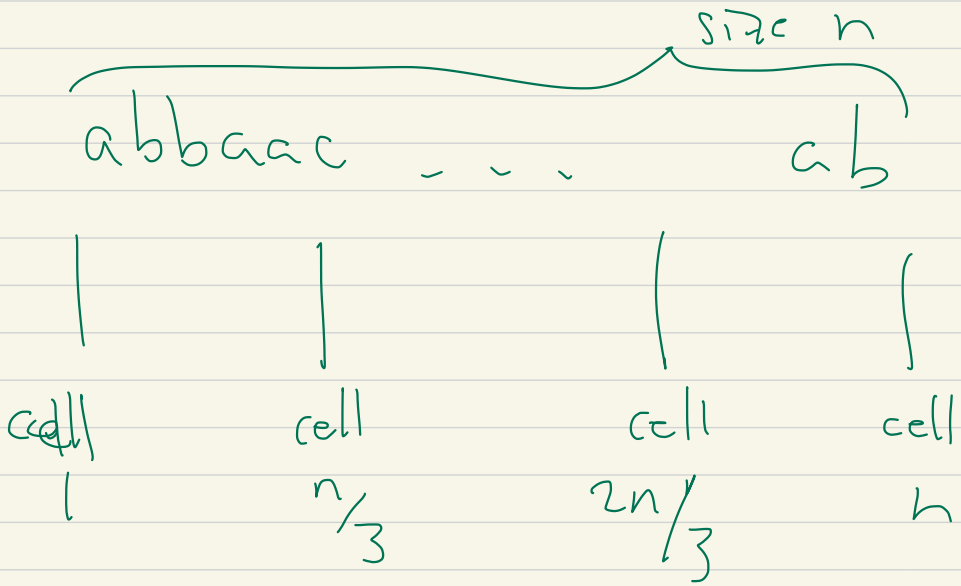
Thm: Any 1-tape algorithm to decide (recognize) $PALINDROME_\Sigma$, $|\Sigma| \geq 2$ take time, for some $c > 0$

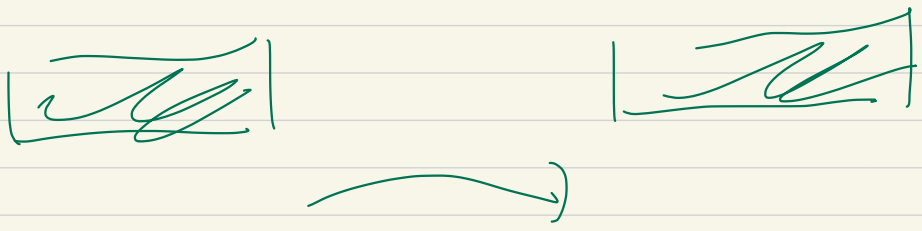
$$\geq cn^2$$

for n sufficiently large

(Not easy, uses "fooling algorithm")

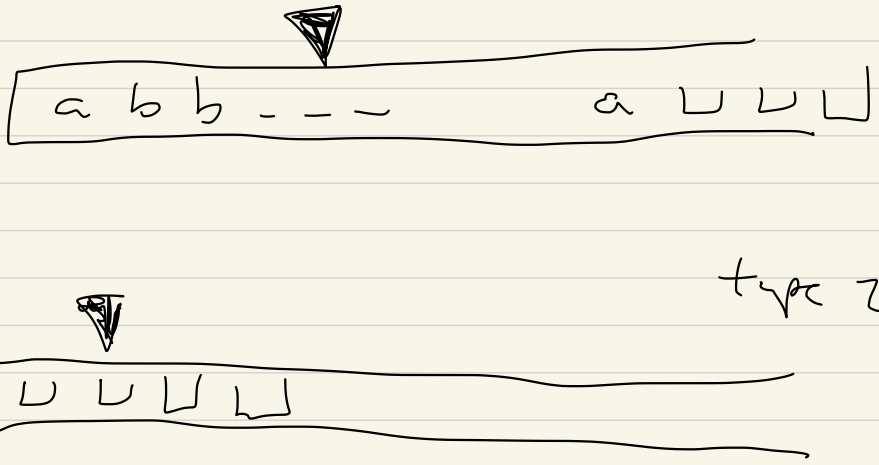
input w, $|w| = n$





TM must make at least c'n jumps back and forth

2-tape TM:



Algorithm:

$$\delta : Q \times \Gamma^2 \to Q \times \Gamma^2 \times \{L, R, S\}^2$$

↑ 2 tape head cells to read

S = stay

Claim: PALINDROME has linear time alg on 2-tape machine

Alg to be described in more detail next Tuesday

10:09 — 10:14 break

Questions related to midterm

Avoid 1,2 concern

C_k for $k=4$

$k=5$

Homework solutions

Min # states

$a^{20}, a^{50} \in L$

$a^{51}, a^{52} \notin L$

=

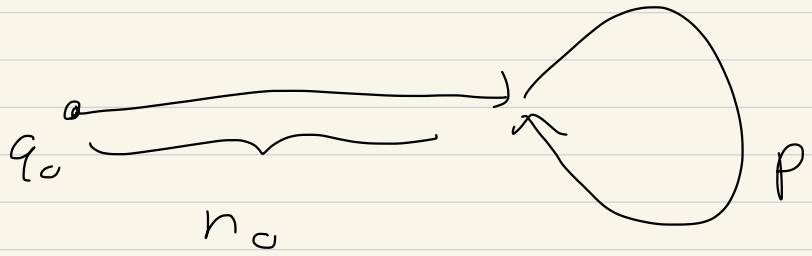
Here

L could be

$$\{ a^n \mid n \bmod 3 = 20 \bmod 3 = 2 \}$$

L is infinite! $\Sigma = \{a\}$

DFA:



L infinite $\Longleftarrow$

periodic part L_{cs}

≥ 1 accepting state

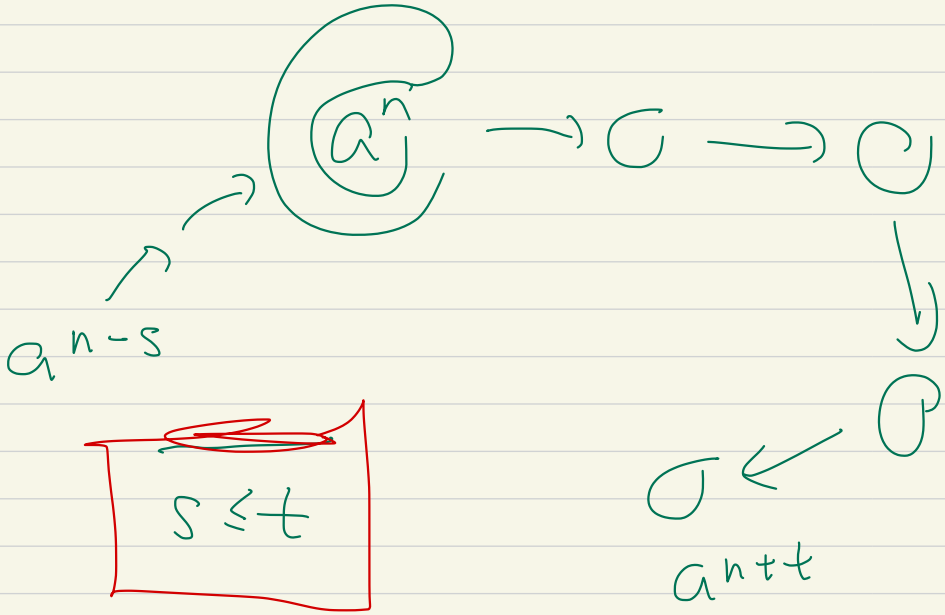
converses

L finite $\Longrightarrow$

every state of period part is not accepting

$a^n \in L$, a^n is taken to an accepting state of any DFA recognizing L

$a^{n+1}, \ldots, a^{n+t} \notin L$



So

Case 1 a^{n-5}, a^n lie on non-per part

Case 2 a^n lies on per. part

a^{n-5} does not

Case 3 a^{n-5}, a^n both lie on periodic part

You are allowed

1 double-sided page of notes,

$8\frac{1}{2}'' \times 11''$

(will also help you to study)