CPSC 421/501 Oct 14, 2021

Think of Ch 1 [Sip]:

§1.1 DFA's

§1.2 N~~F~~A's

§1.3 Regular Expressions

§1.4 Non-regular languages

This year:

Extra material on $\Sigma = \{a\}$

§1.3 only do $\frac{1}{2}$ (the $\frac{1}{2}$ that is useful for applications)

§~~1.4~~ ← not Pumping Lemma

but Myhill-Nerode

Languages!

- Languages over $\Sigma = \{a\}$
- DIV-BY-3

3 state DFA:

$$\{\varepsilon, 3, 6, 9, 03, 06, 09, 12, \ldots\}$$

3 state DFA needed!

$$\{3, 6, 9, 12, 15, \ldots\}$$ exclude ε
exclude leading 0's

- Problems 1.60, 1.61 [Sip]

C_k, $k = 1, 2, 3, \ldots$
$k \in \mathbb{N} = \{1, 2, \ldots\}$

$C_k := \{ w \in \{a, b\}^* \mid$ the k^{th} last symbol of w is an "a" $\}$

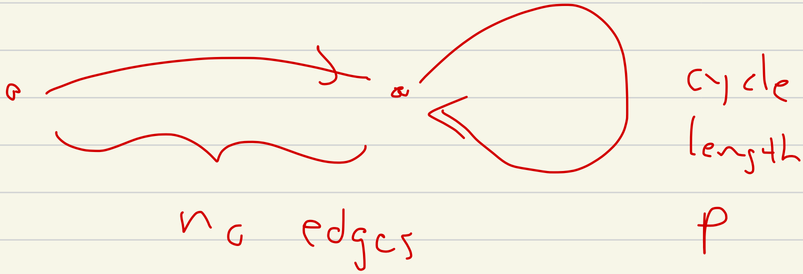
=

$\Sigma = \{a\}$:

$\{ a^n \mid n \text{ is a prime} \}$
$\{ a^{(n^5)} \mid n \in \mathbb{N} \}$
non-regular languages

We know DFA on $\{a\} = \Sigma$

looks like



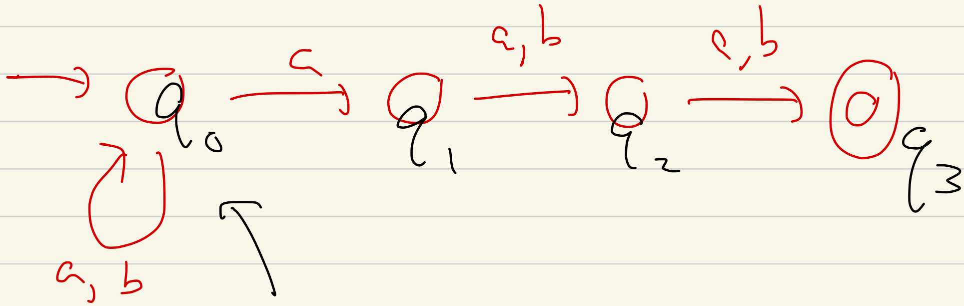
Example of NFA

$\rightsquigarrow$ DFA

e.g.

$$C_3 = \{ w \in \{a, b\}^* \mid \text{the } 3^{rd} \text{ to last letter is an } a \}$$

NFA $Q = \{q_0, q_1, q_2, q_3\}$



DFA!

states = POWER(Q) $= \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_3\}, \{q_0, q_3\} \ldots\}$

start at $\{q_0\}$

=

Start at $\{q_0\}$ initial state

read a → $\{q_0, q_1\}$

~~read~~ b → $\{q_0\}$

read a

$\{q_0, q_1, q_2\}$

read b

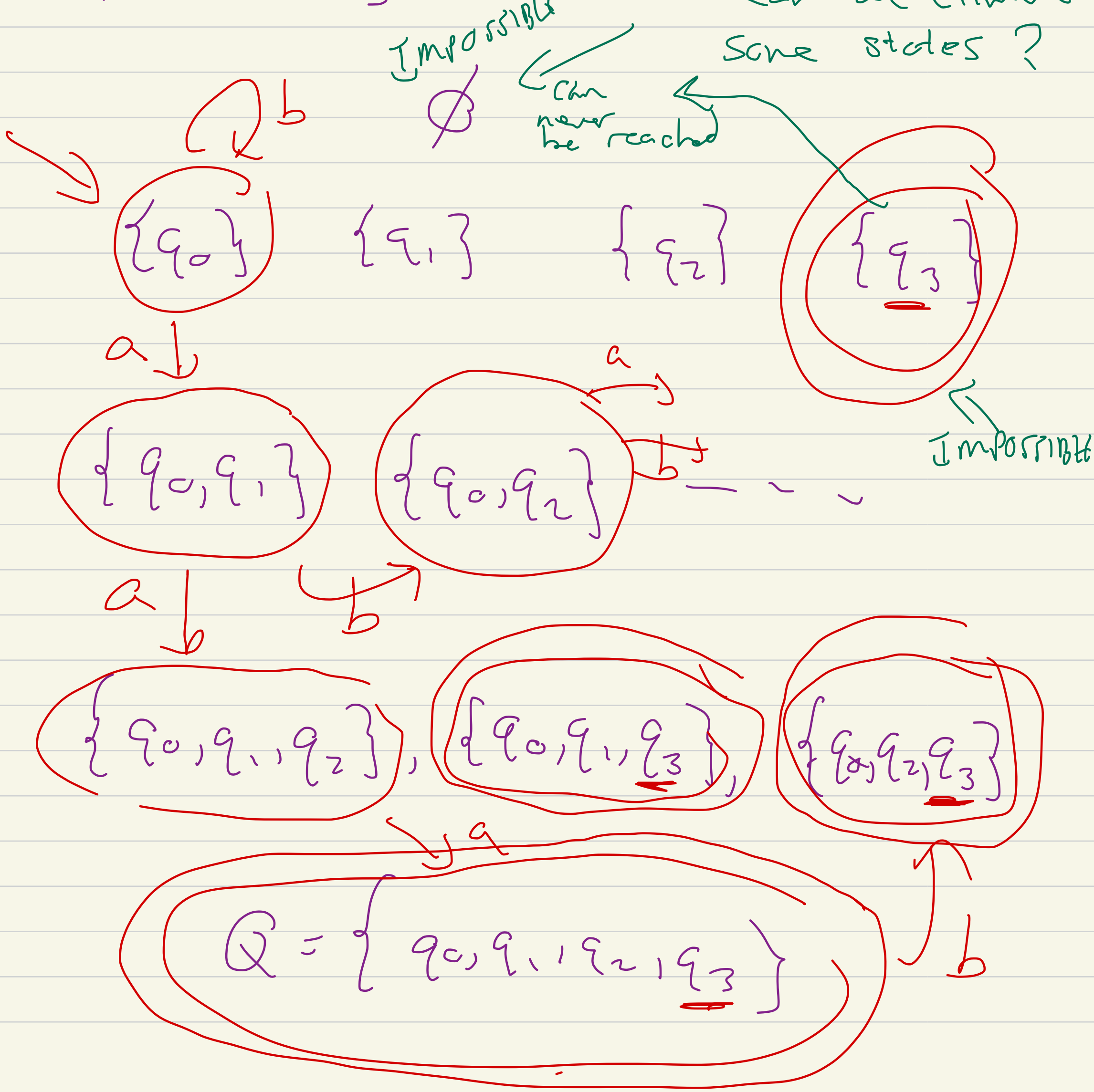
$\{q_0, q_2\}$

read a

read b

State set for DFA has a priori

16 states:



An accepting state here is one that contains q_3, here the only accepting state

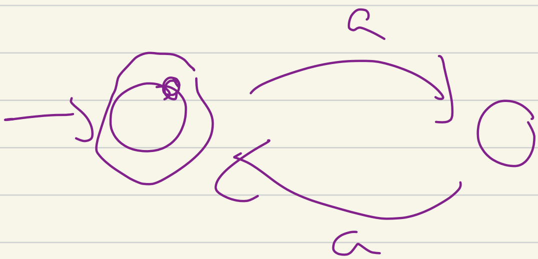
i.e.

Set of final/accepting states of DFA are those subsets that contain q_3

$\{q_0, q_1, q_2, q_3\}, \{q_3\}, \{q_1, q_3\}, \ldots$

e.g., $\{a^n \mid n \in \mathbb{Z}_{\geq 0} = \{0, 1, \dots\}$ s.t. n is even$\}$

$n = 0, 2, 4, \dots$, $\{a^0, a^2, a^4, a^6, \dots\}$



Say input to 16 state DFA is

$\underbrace{b \cdots b}_{7} aaa$ or $b^7 a^3$

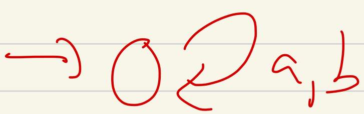
aab _ _ _

?

break 10:11 – 10:16

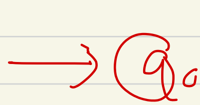
What about ε jumps??

e.g. $L = \emptyset$

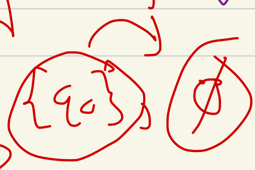


DFA

also



NFA



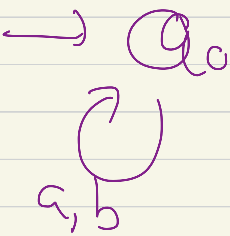
DFA: states

Remark: In above 16 state DFA,

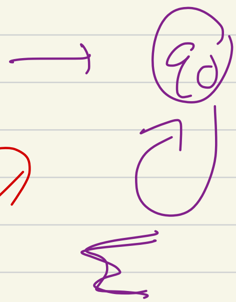
is $\emptyset$ ever reached? No

is $\{q_3\}$ " " ? No

Remark:



or



You can always reach q_0

Gives 2nd reason why

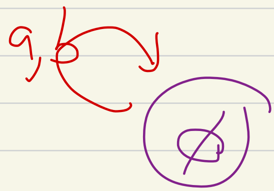
In $C_3 = \{ w \in \{a,b\}^* \mid$ the 3rd to last symbol is an "a" $\}$

there is always some "accepting future"

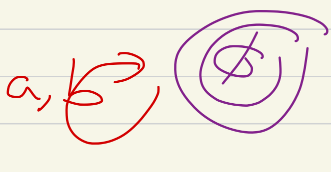
abbbaabababb bbaaa

bbbbb

if we reach $\emptyset$ then either



not accepting



accepting

Q: Can $\emptyset$ be accepting, or reject and reached

<u>Remark</u>: The reasoning here is a warm up for the Myhill-Nerode theorem.

Given a language L, over Σ (alphabet), for any partial input / word / string $w \in \Sigma^*$

$AccFut_L(w)$ "accepting futures of L starting with w"

is defined to be

$$\mathrm{AccFut}_L(w) = \{ u \in \Sigma^* \mid wu \in L \}$$

Example:

$$C_3 = \{ w \in \{a,b\}^* \mid \text{w's 3rd to last symbol is an a} \}$$

$$w = ba$$

$$\mathrm{AccFut}_L(w) = \{ u \in \Sigma^* \mid bau \in L \}$$

concatenation

$$ba \circ \varepsilon = ba \notin C_3$$

$$ba \circ aa = baaa \in C_3$$

$$ba \circ a = baa \notin C_3$$

$$\mathrm{Accfut}_L(ba) = \{ u \in \{a,b\}^* \text{ s.t. } bau \in C_3, \text{ i.e., the third to last symbol of } bau \text{ is an "a"} \}$$

$AccFut_L(ba)$ contains

aa, aaa, a^n for $n \geq 2$

[not a^0 or a^1]

also C_3 itself is there...

b a a

3rd to last symbol is an "a"

then is also works...

anything of length 2,

$\underbrace{b\,a\,\sigma_1\,\sigma_2}_{u}$ is in there...

$\sigma_1, \sigma_2 \in \{a, b\}$

(regex)

$=$

$\mathrm{Accept}_{C_3}(\cancel{b}a) = \Sigma^2 u\, C_3$

anything else?

ba _ _ any 2 symbols

then

is the third to last

Σ^0, Σ^1 don't fall into

$\mathrm{Accept}_{C_3}(ba)$

if

$u \in \Sigma^3 \longrightarrow a\_\,\_$

$u \in \Sigma^4 \longrightarrow \Sigma a \Sigma \Sigma$

Claim:

$$\text{Accfut}_{C_3}(ba) = \Sigma^2 \cup C_3$$

$$\text{Accfut}_{C_3}(\varepsilon) = C_3$$

$\vdots$

$$C_3 \longleftrightarrow \Sigma^* a \Sigma \Sigma$$

After class

Σ = language = $\{a, b\}$

$\Sigma^* = \{a,b\}^*$ = any string

$$\Sigma^* \circ a \circ \{a,b\} \circ \{a,b\} = C_3$$