CPSC 421/501 Oct 12

More surprises ruined ... in Ch. 1

Thm: If L_1, L_2 are regular languages over Σ, then so are:

(1) $\Sigma^* \setminus L_1$ } We flip accepting & non-accepting states Section 1.1 (§1.1)

(2) $L_1 \cup L_2$ } Could do in §1.1, Nice example of NFA (§1.2)

(3) $L_1 \cap L_2, L_1 \setminus L_2$, etc.

(4) $L_1 \circ L_2, L_1^*$ } Easily done with NFA §1.2

(5) L^{rev} } "reverse language" §1.3 (regex)

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§1.2 Thm: Any NFA has a corresponding "efficient" algorithm and a DFA

§1.3

Thm: Any regex (regular expression) has a corresponding DFA or NFA [and vice versa]

Thm: Any DFA recognizing a language L has at least

$$\left|\left\{ \text{AccFut}_L(w) \mid w \in \Sigma^* \right\}\right|$$

states, and conversely

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use this to show:

- some languages non-regular
- min # states for regular lang.

Myhill-Nerode Theorem

Myhill-Nerode Theorem also in 2 exercises of [Sip]

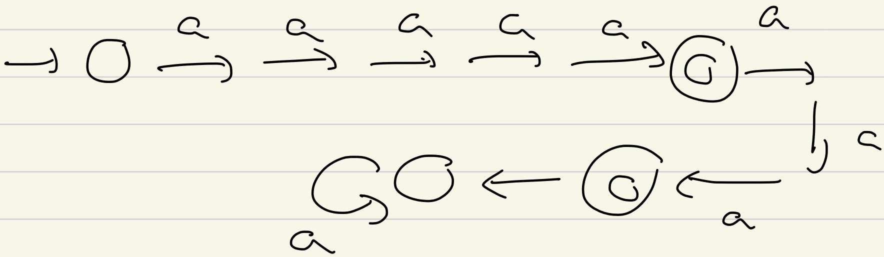
What is an NFA, or

Non-deterministic F.A.?

e.g.

$$L = \{a^5, a^8\}$$

DFA:



Claim: L^* is also regular.

L^* = complicated ...

$$
\begin{aligned}
&\begin{pmatrix}0 \bmod 5\end{pmatrix} && \{\, a^0, a^5, a^{10}, a^{15}, \ldots, \\
&\begin{pmatrix}3 \bmod 5\end{pmatrix} && a^8, a^{13}, a^{18}, a^{23}, \ldots \\
&\begin{pmatrix}1 \bmod 5\end{pmatrix} && a^{16}, a^{21}, a^{26}, \ldots, \\
&\begin{pmatrix}4 \bmod 5\end{pmatrix} && a^{24}, a^{29}, a^{34}, a^{39}, \ldots \\
&\begin{pmatrix}2 \bmod 5\end{pmatrix} && a^{32}, a^{37}, \ldots \,\}
\end{aligned}
$$

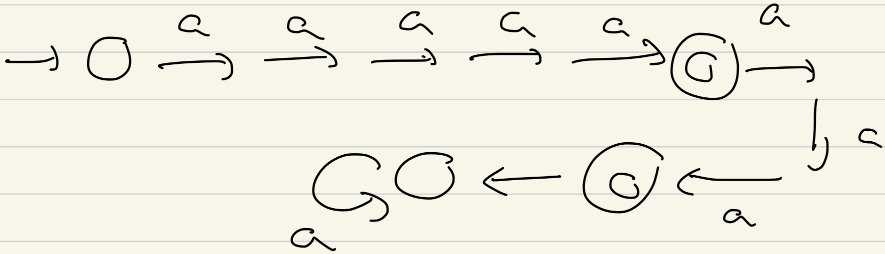
$$= \{ a^0, \ldots \quad \ldots, \underbrace{a^{38}, a^{39}, a^{40}, a^{41}, \ldots}_{\text{stabilizes}}$$

for which min no is

$a^n \in L = \{a^5, a^8\}^*$ for all $n \geq n_0$?

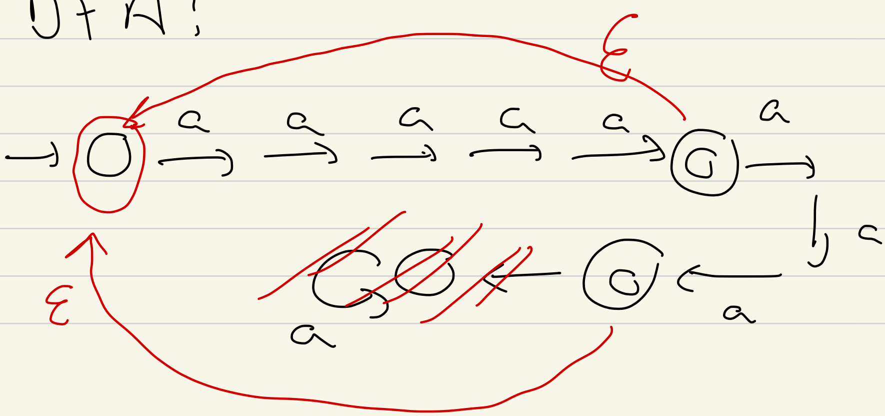
$L = \{a^5, a^8\}$

DFA:



$L^* = \{a^5, a^8\}^*$

DFA:

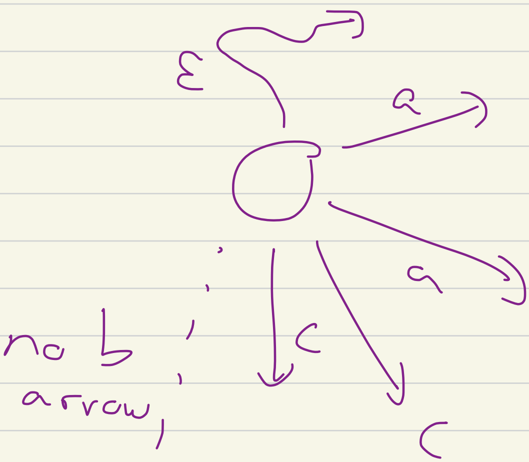


NFA is simple to describe with a diagram:

DFA but also

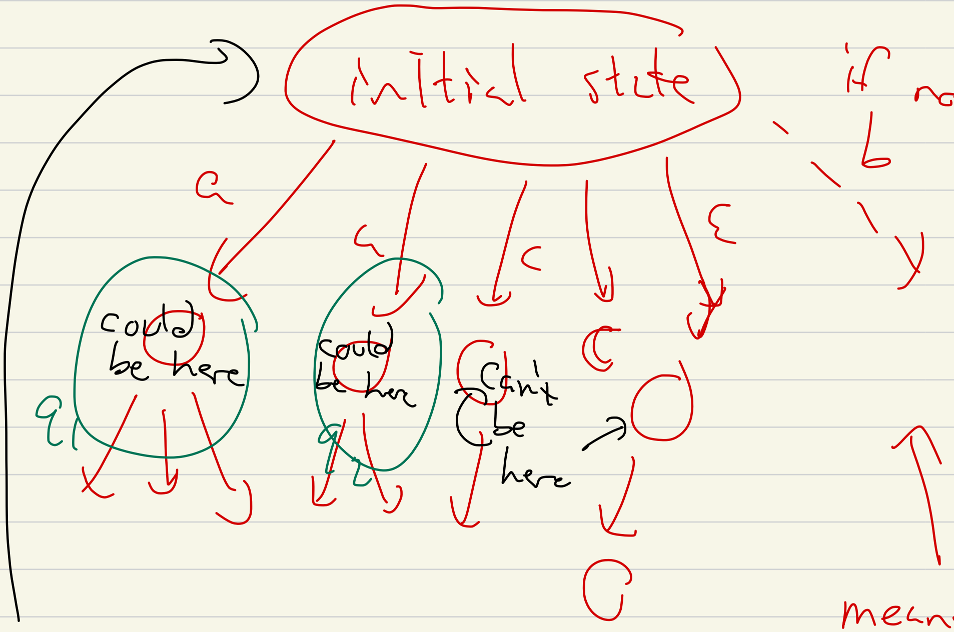
(1) we have an "ε jump"

(2) we allow more than one (or zero) way(s) to leave a state



$\Sigma = \{a, b, c\}$

Way to think of it!



if no b

means you refuse any path that has no way to continue

Input string/word is

abbb...

start! initial state $\xrightarrow{\text{read } a}$ $\{q_1, q_2\}$ $\xrightarrow{\text{read } b}$

Formal definition in §1.2:

$$DFA = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma \to Q$$

$$\cancel{D}\,NFA = (Q, \Sigma, \delta, q_0, F)$$

$$\delta : Q \times \Sigma_\varepsilon \to \mathrm{POWER}(Q)$$

$$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$$

else have ε jump

$POWER(Q) =$

$\{$ all subsets of $Q \}$

Thm: For each NFA with m states, there is a DFA with at most 2^m states that recognizes the same languages.

Let NFA $= (Q, \Sigma, \delta, q_0, F)$.

Let DFA $= ($?? $, \Sigma,$

5 minute break 10:12 → 10:17

6.1.5 constains

fix after class !!

Thm: For each NFA with m states, there is a DFA with at most 2^m states that recognizes the same languages.

Let NFA $= (Q, \Sigma, \underline{\delta}, q_0, F)$.

Let DFA $= \left(\text{Power}(Q), \Sigma, \tilde{\delta}, \{q_0\}, \tilde{F}\right)$

$\tilde{\delta}$: Power$(Q) \times \Sigma \longrightarrow$ Power(Q)

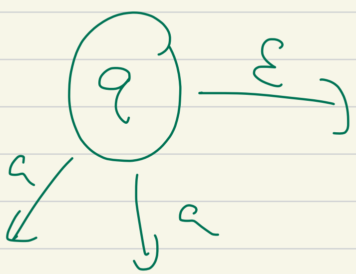
$\tilde{\delta}(S, \sigma) = \{ q$ s.t. $q = \delta(q', \sigma)$ with $q' \in S$

messy ---

$S \subseteq Q$

$S \in Power(Q)$ means $S \subset Q$

$= \{q_{i_1}, q_{i_2}, \ldots\}$



$\hat{F} \subset POWER(Q)$

$\hat{F} = \{ S \subset Q$ s.t.

S contains an element of F,

$S \cap F$ is non-empty, there is a $q \in Q$ s.t. $q \in S$ and $q \in F$

Question: Note (homework) that a DFA with m states can have roughly $2^m/2$ states (certainly $\Omega(2^m)$, i.e. $\geq 2^m/c$)

=

$L = \{ w \in \{a,b\}^* \mid$ the mth last symbol of w is $a \}$

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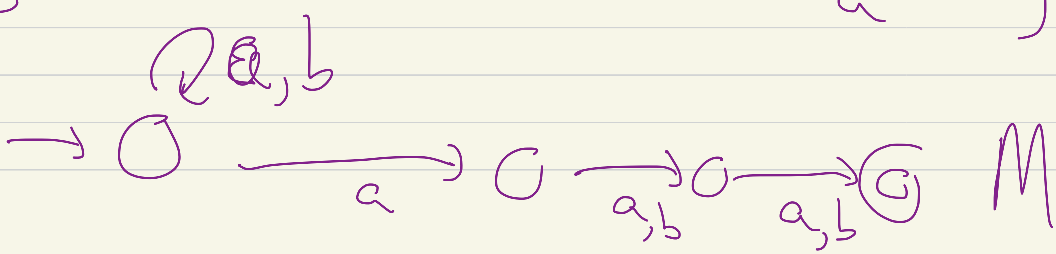
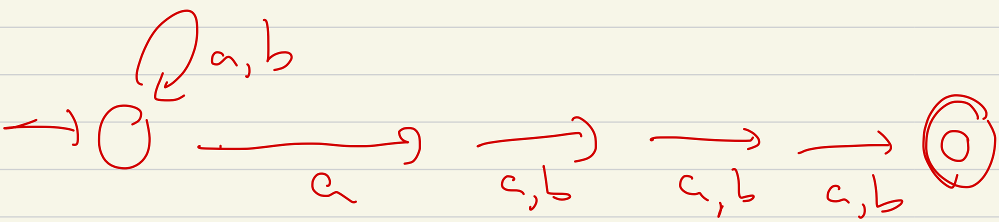


M is an NFA that recognizes

$L = \{ w \in \{a,b\}^* \mid w$ ends in $a\{a,b\}\{a,b\} \}$

i.e. the 3^{rd} to last symbol of w is an a.

Similarly for



4^{th} to last symbol of w is a

Rem:

If $w \in \Sigma^*$, Σ alphabet

$w = \sigma_1 \ldots \sigma_n$ $\quad \sigma_i \in \Sigma$

then

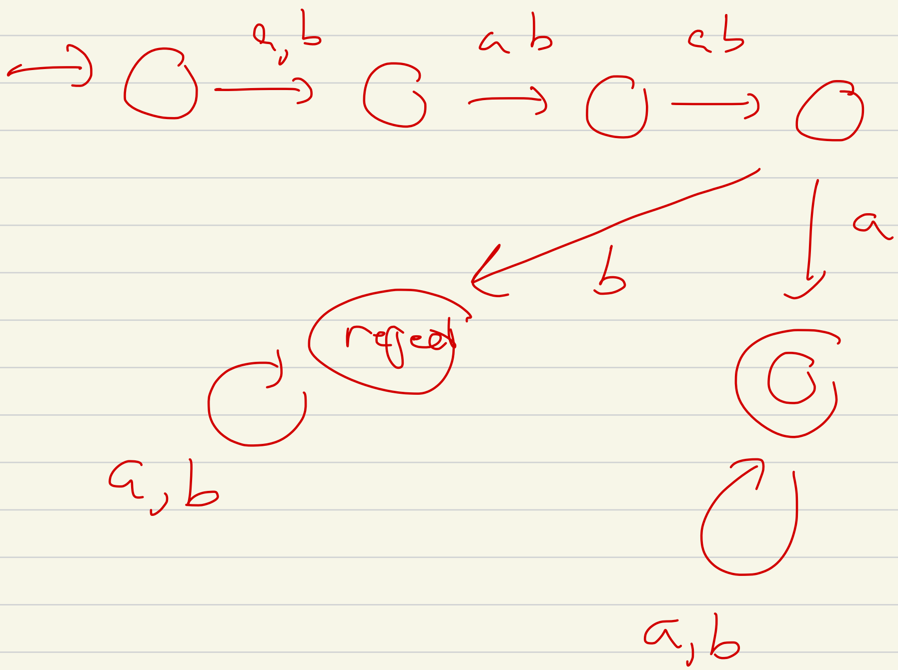
$w^{rev} = \sigma_n \sigma_{n-1} \ldots \sigma_1$

$$L = \{ w \in \{a,b\}^* \mid \text{the 4th to last symbol of } w \text{ is } a \}$$

$$L^{rev} = \{ w \in \{a,b\}^* \mid \text{4th symbol (from beginning) is an } a \}$$

Is there a short DFA for

L^{rev} ?



RegEx for L!

... Σ 1, 3 ...