CPSC 421/501 Oct 5

Sept 28: [Sip], Section 1.1

{Regular languages} = non-neg langs

{Languages recognized by a DFA}

"Complexity" := minimum # of states

Theorem: If L_1, L_2 are regular then

$L_1 \cup L_2$, $L_1 \cap L_2$ are regular (§1.1)

$L_1 \circ L_2$, L_1^* are regular need (§1.2)

Section 1.2: NFA's (non-deterministic)

$L = \{a^6, a^{14}\}^* = \{\varepsilon, a^6, a^{12}, a^{14}, \dots\}$

how to build a DFA for L ????

Question:

What are the advantages of each of the alphabets below over the others?

① $\Sigma = \{a\}$ ← fundamental in that it seems simple, basic algorithms

② $\Sigma = \{a, b\}$ useful in examples

(TODAY)

③ $\Sigma = \{0\}$ Do not underestimate (sneer) at unary

④ $\Sigma = \{0, 1\}$ suggest binary notation

⑤ $\Sigma = \{0, 1, \ldots, 9\}$ suggests decimals (TODAY)

(Today there are one or two main points, for Chapter 1 ...)

Recall: If $L \subset \Sigma^*$,

Σ = alphabet, then

$$L^* = \{ w_1 \circ w_2 \circ \dots \circ w_k \mid \text{each } w_i \in L \}$$

words/strings $\quad \circ$ concatenation

$$aba \circ ba = ababa$$

$$L_1 \circ L_2 = \{ w_1 \circ w_2 \mid w_1 \in L_1,\ w_2 \in L_2 \}$$

$$aba \circ ba$$

$$(a, b, a) \circ (b, a)$$

$$= (a, b, a, b, a)$$

$$\{a, bb\} \circ \{c, dd\}$$

$$= \left\{ \begin{array}{ll} ac, & add, \\ bbc, & bbdd \end{array} \right\}$$

$\because L$

PRIME Power of a $= \{a^2, a^3, a^5, a^7, \ldots\}$

Goldbach conj:

any even number $\geq \cancel{4}\ 6$ can be written as sum of 2 odd primes.

I.e.,

Correction after class here!

$L^{odd} = \{a^3, a^5, a^7, a^{11}, \ldots\}$

$L^{odd} \circ L^{odd} = \{\cancel{a^4}, a^6, a^8, a^{10}, a^{12}, \ldots\}$

e.g. $a^{10} = a^5 \circ a^5 = a^3 \circ a^7 = a^7 \circ a^3$

My thoughts...

- Information capacity in bits
- Σ^* all countably infinite
- etc.

SURPRISE! UNARY

- Useful in Chapter 1, e.g. to illustrate $*$, e.g. $L = \{a^5, a^9\}$, $L^* =$ 
- Gives NP-complete problem SNEAK-NP almost immediately

Policy:

1. Model homework solutions will be presented as solutions to class with name(s) not displayed unless you opt out, with:

- "Not To Be Used As An Exemplary Solution"
- "Leave Name if Selected"

2. Office hour Piazza poll

$\Sigma = \{a\}$, so $|\Sigma| = 1$

$\Sigma^* = \{\varepsilon = a^0, a^1, a^2 = aa, a^3, \ldots\}$

$\Sigma^* \longleftrightarrow \mathbb{Z}_{\geq 0} = \{0, 1, \ldots\}$

$a^m \longleftrightarrow m$

Easy to describe &

(1) regular vs. non-regular

(2) lower bounds on how many states needed in a DFA to

recognize $L \subset \Sigma^* = \{a\}^*$:

Really!

$$DFA = (Q, \Sigma, \delta, q_0, F)$$

states

alphabet

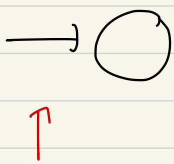
transition function

initial state

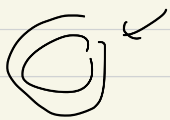
accepting (final) states

$$\delta : Q \times \Sigma \to Q$$

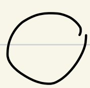
Notation:



initial state



final/accepting state

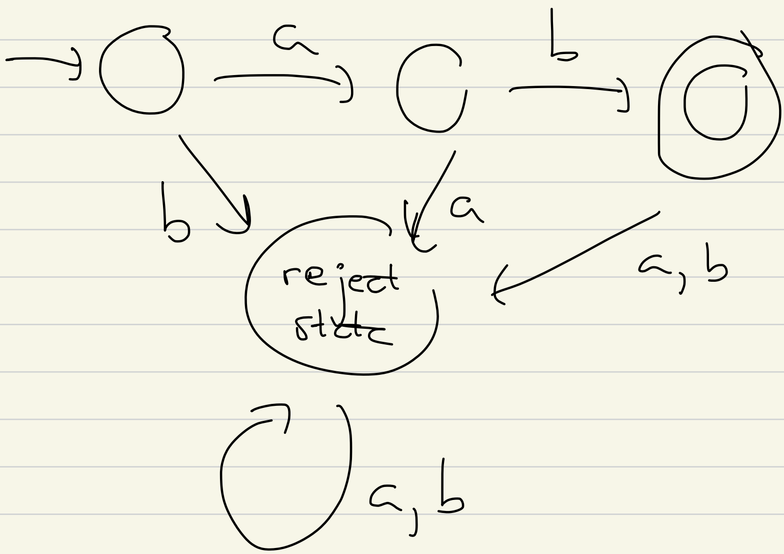


not an accepting state

Example:

$$L = \{ab\} \in \Sigma^* = \{a, b\}^*$$

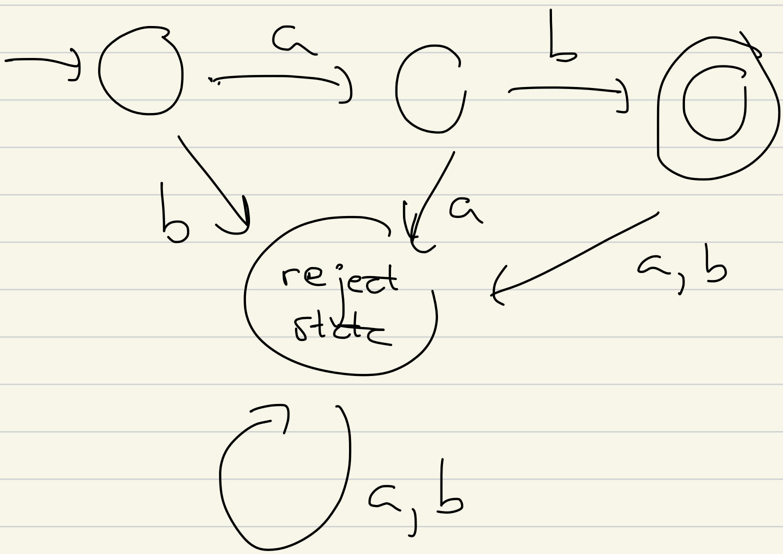
$$|L| < \infty, \quad |L| = 1$$



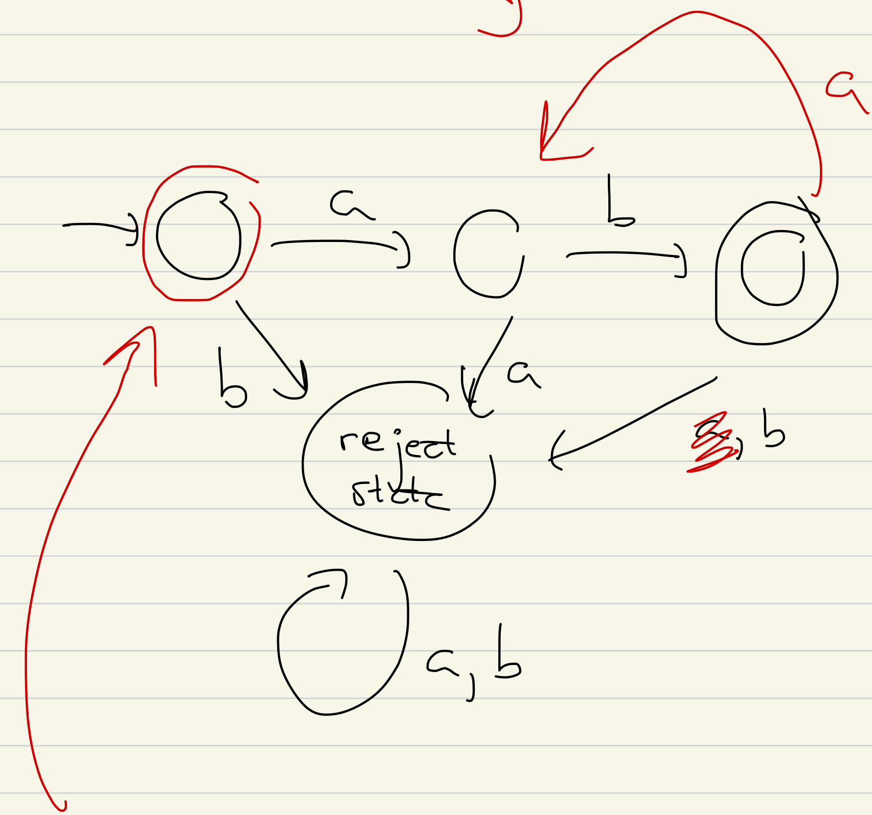
$$L = \{ ab \}$$

$$L^* = \{ab\}^* = \{\epsilon, ab, abab, ababab = (ab)^3, (ab)^4, \dots\}$$

Modify: (how?)



Modification 1



Modification 2: accept the empty string

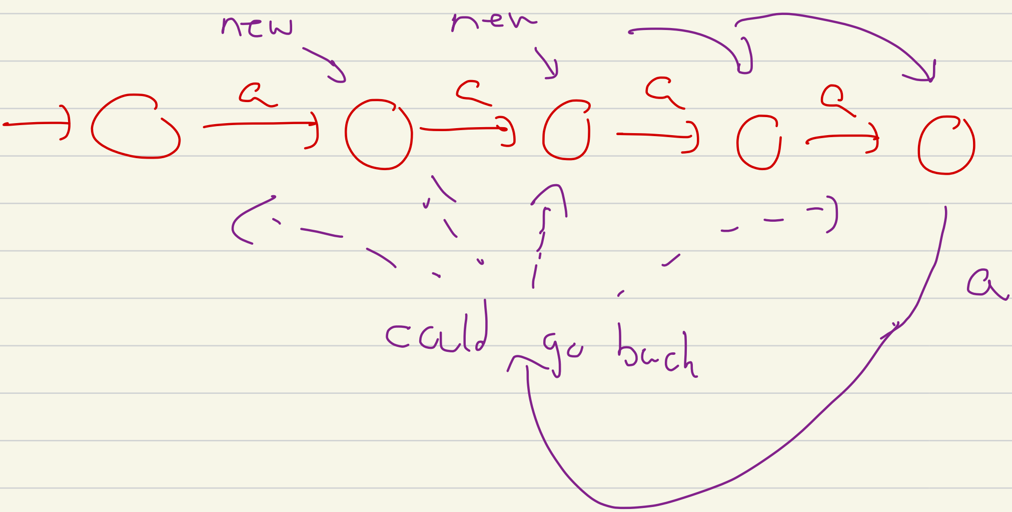
break 5 min 10:10–10:15 am

(1) Languages over $\Sigma = \{a\}$

Don't sneer at $|\Sigma| = 1$

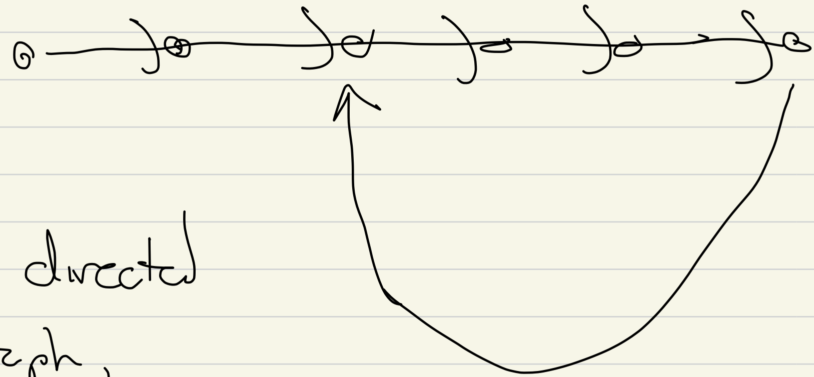
(2) DIV-BY-3

DFA's over $\Sigma = \{a\}$



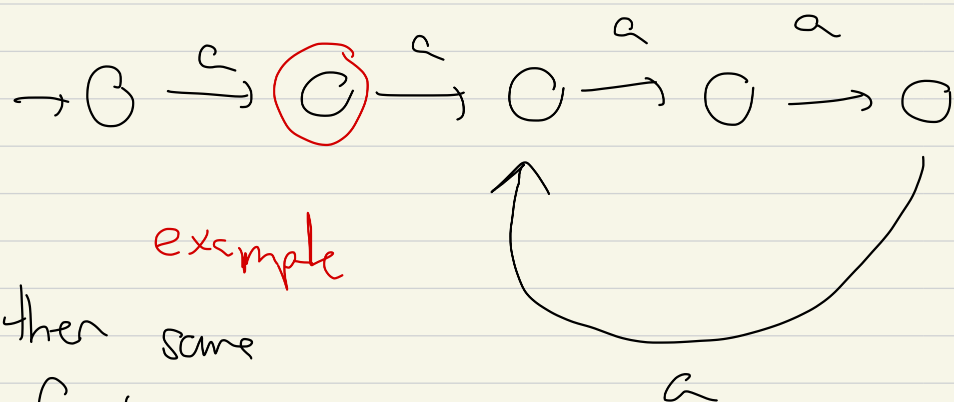
DFA over $\Sigma = \{a\}$

e.g.,



a directed graph, outdegree 1

OR

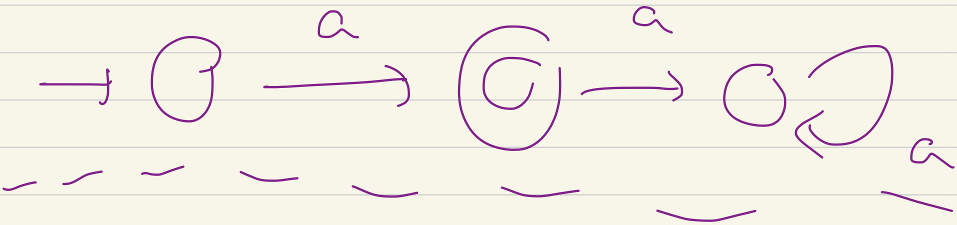


example

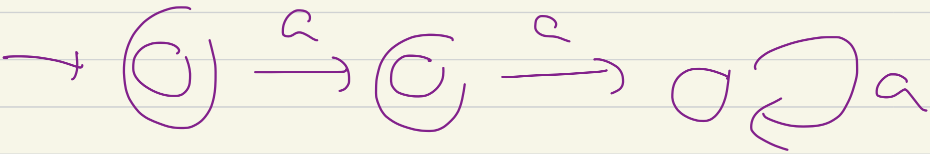
then some final states

this recognizes $L = \{a\}$

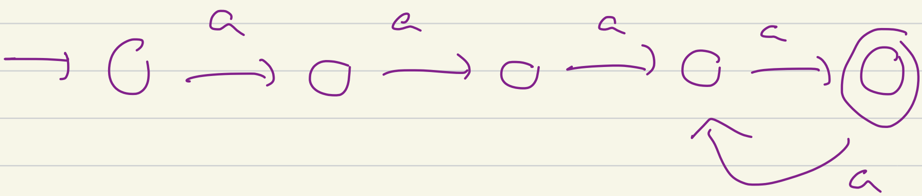
also



or



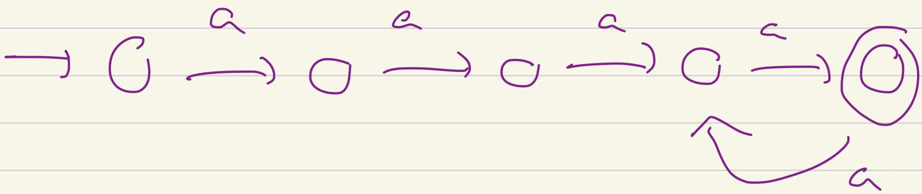
recognizes $L = \{\varepsilon, a\}$



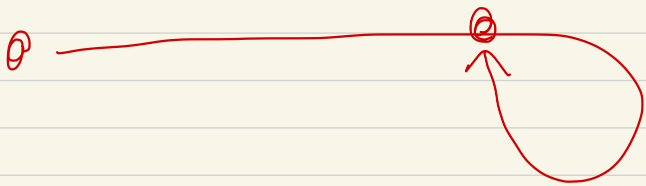
recognizes

$$= \{ a^4, a^6, a^8, \ldots \}$$

$$= \{ a^n \mid n \text{ is an even integer} \geq 4 \}$$



periodicity = 2



length of loop

Definition: Say that $L \subset \Sigma^*$

$\Sigma = \{a\}$ is eventually periodic

if $\exists\, n_0 \in \mathbb{Z}_{\geq 0}$ and

$p \in \mathbb{N} = \{1, 2, \ldots\}$ s.t.

$\forall n \geq n_0$

$$a^n \in L \underbrace{\iff}_{\text{if and only if}} a^{n+p} \in L$$

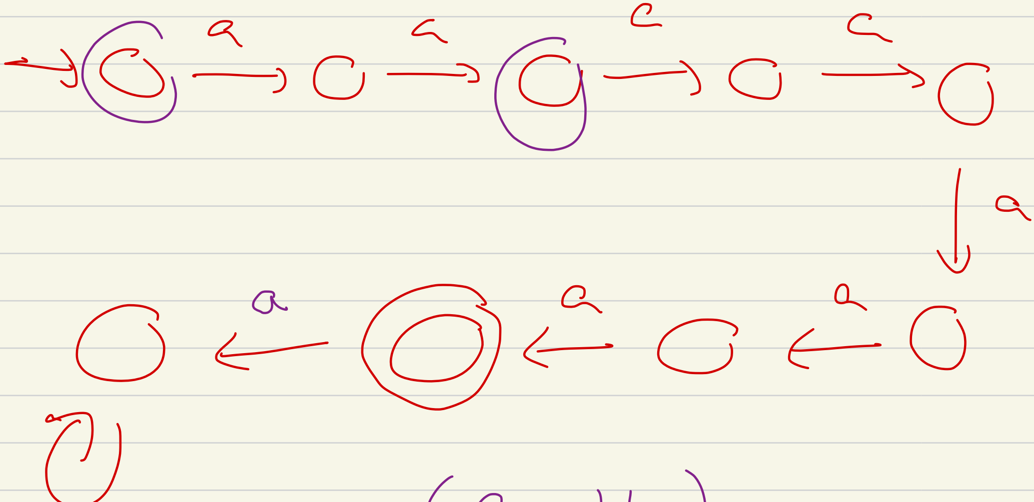
Alternatively: for sufficiently large

n we have

Rem: say $L \subset \{a\}^*$ is finite

$L = \{\varepsilon, a^3, a^7 \}$.

What is smallest number of states needed in a DFA recognizing L?



(9 states)

↑
Stuff appears on homework
=

DIV-BY-3

$= \{ n \in \{0, 1, \ldots, 9\}^*$ s.t.

n is divisible by 3 $\}$

= there are a few possibilities

① DFA with 3 states can

recognize:

$\{ \varepsilon, 0, 3, 6, 9, 03, 06, 09,$

$12, 15, 18, 21, \ldots \}$

$\{ 0, 3, 6, 9, \quad \}$

other extreme

DIV-BY-3

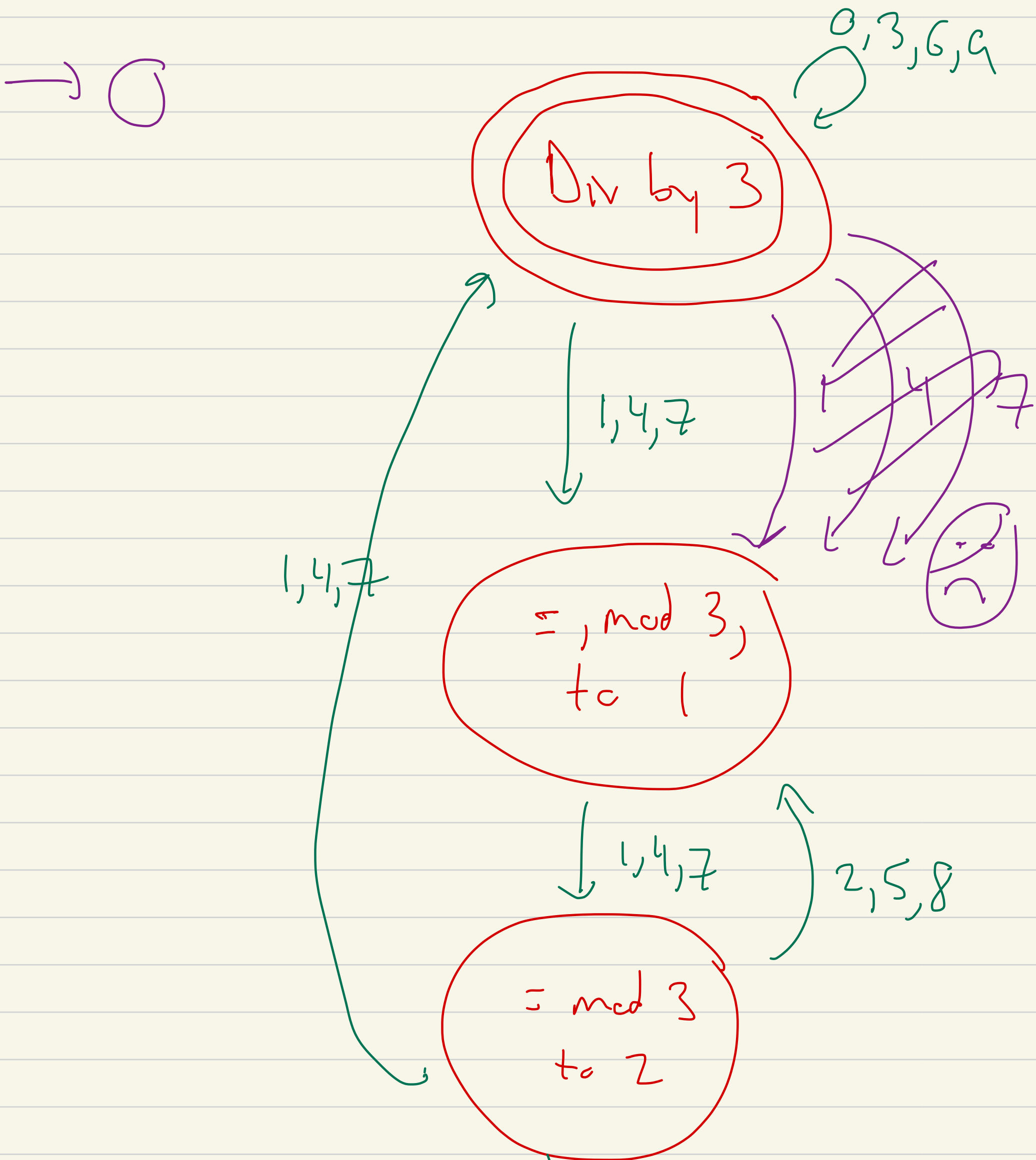
"keep it real"

$\{ 3, 6, 9, 12, 15, 18, \ldots \}$

takes 5 states, no ε

no leading 0's

$\Sigma = \{0, 1, \ldots, 9\}$



to be contined next time