

CPSC 421/501, Sept 23, 2021

- We will return to more facts about countability, etc. when we discuss §4.2 of [Sip]. For now:

- Appendix A of Handout

Alphabets, Strings, etc.



(Chapter 0 of [Sipser])



- Appendix B of Handout

B.1: Injections, ... (Ch 0 [Sip])

B.2 : Countable Sets

§4.2 Section 4.2 [Sip]

B.3: Cantor's Theorem

TODAY!

||

I have

learned

1 syllable

We are

learned

2 syllables

Countability!

- finite sets Appendix A, B
Ch. 0 [Sip]

- Countably Infinite

e.g.,

$$\mathbb{N} = \{1, 2, 3, \dots\}$$

$$\text{ASCII}^* = \text{as a set} \quad \Downarrow$$

$$\{a\}^* = \{\epsilon, a, a^2, \dots\}$$

} all
the
same
size

Problem 8.3.5

Johnny

Maria

Alexis

David

$S, |S| = 3$

$S = \{$ Prof. Hummos
Prof. Humos,
soft
Prof. Pita,
Prof. Halva

X { (with is good) (loves) } Y

Y (foods)

Humus	Pita	falafel
-------	------	---------

Prof. Humus
 X Prof. Pita
 Prof. falafel

no	yes	yes
no	yes	no
yes	yes	yes

TABLE 1

Binary relation

yes

no

no

$T = \{ \text{humus}, \quad , \quad \}$

$\mathcal{P} =$ profs
pigeons
programs

$\mathcal{I} =$ ice creams
inputs

$\mathcal{P} = \mathcal{I}$ (after \mathcal{I} fix set)

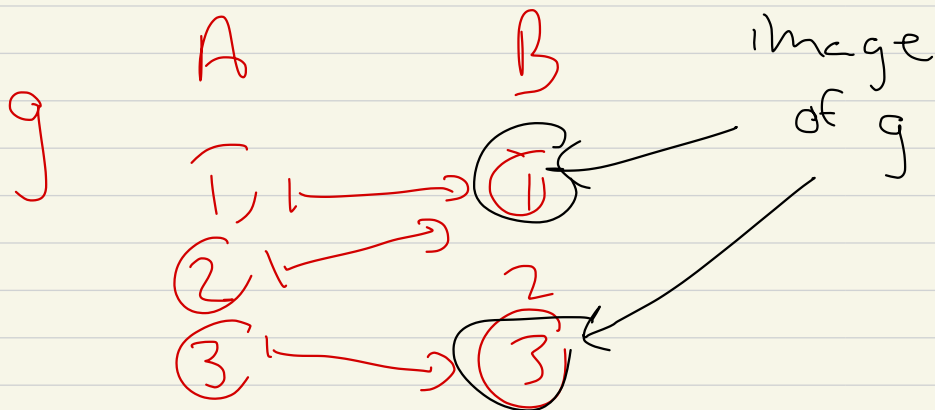
Cantor's Thm!

Let $f: S \rightarrow \text{Power}(S)$. Then

$$T = \{ t \in S \mid t \notin f(t) \}$$

Then T is not in the
image of f .

If $g: A \rightarrow B$ is a
map of sets, then the
image of g



Prof Humus loves

{ , pita, folefal }

$T \neq$ Prof Humus loves

↓

{ humus, - }

↓

{ no humus, - }

⊆

$S \rightarrow \text{Power}(S)$

$|S| = 3$

↑

size

size 3

size $2^3 = 8$

5 minute break

10:06 - 10:11



$f: S \rightarrow \text{Power}(S)$

Prof Humus $\mapsto \{ \text{Pita}, \text{Falafel} \}$

Prof Pita $\mapsto \{ \text{Pita} \}$

Prof Falafel $\mapsto \{ \text{Humus}, \text{Pita}, \text{Falafel} \}$

$S = \{ \text{Humus}, \text{Pita}, \text{Falafel} \}$

Why diagonal?

Not so important

Generating an equivalent table:

with is good loves

Y (foods)

	Falafel	Pita	Humus
Prof. Humus	no	yes	yes
Prof. Pita	no	yes	no
Prof. Falafel	yes	yes	yes

TABLE 2

X {with is good loves} Y

Y (foods)

	Humus	Pita	falafel
Prof. Humus	yes	yes	no
Prof. Pita			
Prof. falafel			

Prof. Humus

X {Prof. Pita

Prof. falafel

TABLE 2'

Original Names

A new example via renaming

TABLE 1

	H	P	F
H	no	yes	yes

P
F

How to use
off diagonals to
produce something not

H → { P, F } in the image

$$T = \left\{ \begin{matrix} 1 \\ \cancel{0} \\ \vdots \end{matrix} \right\}$$

P →

[Nice homework problem
for 2021 or
later. All one
needs is a permutation

$$\pi : S \xrightarrow{\text{perm}} S$$

$$f: S \rightarrow \text{Power}(S)$$

$$\pi: S \rightarrow S \text{ permutation}$$

$$T = \{ t \in S \mid t \notin \pi^* f(t) \}$$

$$\pi^* f \text{ (maybe } \pi_* f \text{ ?)}$$

$$\pi^*: \text{Power}(S) \rightarrow \text{Power}(S)$$

should be enough

How many people
(maybe excluding prof)
are there?

Number between

~~28~~ 28 - 31

that is your birthday
date (not) month

med 10

2 special people
=>

~~4~~ + ~~9~~

+ ~~5~~

= ~~8~~ (8)

How many with

$$7 ? = 4$$

$$2 ? = 4$$

step

roughly 40