Q: Why study §4.2 of [5ip] first abstractly, and then again?

There are many answers.

1. “In mathematics you don’t understand things. You just get used to them.”

John(hy) von Neumann, via Warren Dicks’ homepage:

[https://mat.uab.cat/~dicks](https://mat.uab.cat/~dicks)

verified this is the real Warren Dicks
Announcements:

- Office hours posted
  (8 pm office hour Zoom only, will disappear if no one shows up)

- Masks — fully covering nose and mouth when worn indoors

[ - No candy today 😞]

- Truth & Reconciliation: Sept 30
Class answers:

(A) You get used to it, makes you less anxious.

(B) You can be looking for the details you already recognize.

(C) Ender’s Game

Video games
Black Mirror
Olga Todd is an example of (editing) Wilhelm's led.
More of my Answers

2) J.C. talk about losing script and rewriting, 2nd version better than first. Work not really lost...

3) Student who unknowingly learned

4) When I took the same course in the late 1970's, I could do the problem sets, but felt totally confused. And I still do with this same proof.

5) Abstractly, we get to the bare minimum
Klaus Hoechsmann’s approach to teaching introduction (first term, honours, non-honours, etc.)

- 1st 2 weeks: teach entire course for 2×2 systems (2×2 eigenvalues, etc.)

Some textbooks lack sufficient motivation, at least from my point of view, but I don’t have the time to...

First week or two is good for shopping...
GOALS FOR TODAY
- What does $|S| < |T|$ mean for infinite sets?
- Show $I$ undecidable (or even unrecognizable, etc.) problems
- Start "Abstract" undecidability

\[
\text{ACCEPTANCE} \overset{\text{def}}{=} \{ \text{EncodeBoth} (p, i) \mid \text{Result} (p, i) = \text{yes} \}\\
\]
Last time:

Programs $\subseteq$ \textit{ASCII} $^*$

(Profs (Pigeons) $\cap$ word (linguists)

Inputs = \textit{ASCII} $^*$

(Ice Cream)

Languages = \text{Power}(\text{ASCII}^*)

= Set of all subsets of \textit{ASCII} $^*$
Power(S) = 2^S

def set of all subsets of S

Cantor's Theorem:

|S| is smaller than |2^S|

\[ |\text{set of all subsets of } S| \leq |\text{Power}(S)| \]

Finite case:
Idea: If \(|P| < |L|\), then some element of \(L\) is not in the image of \(f\).
What does it mean for infinite set \( P, L \) to say \( |P| < |L| \) strictly less?

1. \( \forall f: P \rightarrow L \)
   Image \((f)\) is not all of \( L \).

2. \( \)
\[ \mathbb{N} = \{ 1, 2, 3, \ldots \} \]

\[ \mathbb{Z} = \{ 0, 1, -1, 2, -2, \ldots \} \]

\( \mathbb{N} \subset \mathbb{Z} \)

\text{fine:}

\[ \mathbb{N} = \{ 1, 2, 3, \ldots \} \]

\[ \mathbb{Z} = \{ \ldots, -4, 0, 1, 2, 3, \ldots \} \]
\[ Q = \{ \text{rational numbers in } \mathbb{R} \} \]

5 is countable

Function: \( f : S \rightarrow T \)

Image of \( f \): \( \text{Image}(f) = \{ t \in T \mid \exists s \in S \text{ s.t. } f(s) = t \} \)

(\text{Corrected from } \emptyset \text{ by Andre R. on Sept 21 2021 Office Hours})
Profs $\rightarrow$ Bird Setn

$\text{Image}(f) = \{1, 2, 5\}$

(Not in image)
\[
\text{Power}(\{1, 2\}) = \big\{ \emptyset, \{1\}, \{2\}, \{1, 2\} \big\}
\]

\[
\text{Power}(\{1, 2, 3\}) = \big\{ \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \big\}
\]
\{1,2\} \text{ same } \{2,1\}

(1,2) \text{ not same } (2,7)

Strings over \text{a,b}: \\
\{ \text{e, (a), (b), (a,a)}, \\
\{ (a,b), (b,a), (b,b) \} \cdots \\
Sequences
typically write
\((a, a, b, a)\)
as \ aaba

Hi there!

\((H, i, U, t, h, e, r, e, !)\)

Chapter 0 of \([Sip]\)
Now Cantor’s Theorem with $3 \times 3$ example

Break

Universal TM, Encode, etc.
A set is countably infinite if it is in bijection with

\[ \mathbb{N} = \{ 1, 2, 3, \ldots \} \]

There exists a sequence

\[ S_1, S_2, S_3, \ldots, S_n \in S \]

and

\[ \{ S_1, S_2, \ldots \} = S \]
Countable = Either Finite or Countably Infinite

Finite ✓

Countably Infinite
for all $n = \{0, 1, 2, \ldots \}$

$g(2n + 1) = n$

for $n = 1, 2, \ldots$

$g(2n) = -n$

$\mathbb{Z} = \{0, -1, 1, -2, 2, \ldots \}$

exhaustive sequence

*Caveat:*
\[ \{1,2\} = \{1,2,2,2,2,\ldots\} \]

\[ \{1,2\} \subseteq \{s_1, s_2, s_3, \ldots\} \]

\[ 1, 2, 2, 2, 2, \ldots \]

**Countable**

2. There exists a sequence

\[ s_1, s_2, s_3, \ldots, s_i \in S \]

and

\[ \{s_1, s_2, \ldots\} = S \]
The retractions are countable

0 - 1 - 1 - 2 - 2 - 3 - ...
Strict bijection

\[
\begin{align*}
0, & \quad 1, & \quad 0/1, & \quad -1, & \quad 0 \\
1, & \quad 1, & \quad 1/2, & \quad 1, & \quad 2/3, & \quad 1
\end{align*}
\]

IN \subset \mathbb{Z} \text{ but}

can be put into

1-1 bijection
All this (are more) in

\[ \text{Sip} \] \ \& \ 4.2

Exercise: Show that there are uncountably many bijections

\[
\begin{array}{cc}
1 & 1 \\
2 & 2 \\
3 & 3 \\
4 & 4 \\
5 & 5 \\
6 & 6 \\
\end{array}
\]
Instable

Uncountable

ASCII

There is no way to describe all bijections \( \mathbb{Z} \rightarrow \mathbb{Z} \) in "words".

Theorem: If \( g \) is a map

\[ g : \text{ASCII} \rightarrow (\text{bijective } \mathbb{Z} \rightarrow \mathbb{Z}) \]

then \( g \) is not surjective,
Image (g) is not all of (bijection $\mathbb{Z} \to \mathbb{Z}$)

\[ f \]

Program $\rightarrow$ Languages

1 $\rightarrow$ 1
2 $\rightarrow$ 2
10 $\rightarrow$ 1

This is not by this notation that $f$ is a function.
So if here is a fixed interpretation.

exhaustive see vs

"..." will distinct elements.