CPSC 421/501 Sept 21, 2021

Q: Why study §4.2 of [Sip] first abstractly, and then again?

There are many answers.

(1) "In mathematics you don't understand things. You just get used to them."

John(ny) von Neumann, via

Warren Dicks' homepage:

https://mat.uab.cat/~dicks

verified this is the real Warren Dicks

Announcements:

- Office hours posted
  (8pm office hour Zoom only, will disappear if no one shows up)

- Masks — fully covering NOSE and mouth when worn indoors

[- No candy today ☹]

- Truth & Reconcilliation! Sept 30

Class answers!

(A) You get used to it, makes you less anxious.

(B) You can be looking for the details you already recognize.

(C) ~~C~~ B — Ender's Game }
" Shadow }

Video games
Black Mirror

{ other Chrome
{ examine code

(D) Olga ... Todd ←
example of (editing
Hilbert's) ~~Collect~~

(E)

(F)

G

H

I

Add page?

MORE OF MY ANSWERS

(2) J.C. talk about losing script and rewritting, 2nd version better than first. Work not really lost...

(3) Student who unknowing learned

(4) When I took the same course in the late 1970's, I could do the problem sets, but felt totally confused. And I still do with this same proof.

(5) Abstractly, we get to the bare minimum

(6) Klaus Hoechsmann's approach to teaching introduction (first term, honours, non-honours, etc.)

- 1^{st} 2 weeks: teach entire course for 2×2 systems (2×2 eigenvalues, etc.)

(7) Some textbooks lack sufficient motivation, at least from my point of view, but I don't have the time to ...

(8) First week or two is good for shopping...

GOALS FOR TODAY

- What does $|S| < |T|$ mean for infinite sets?

- Show $\exists$ undecidable (or even unrecognizable, etc.) problems

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- Start "Abstract" undecidability of

$$\text{ACCEPTANCE} \stackrel{\text{def}}{=} \{ \text{EncodeBoth}(p, i) \mid \text{Result}(p, i) = \text{yes} \}$$

Last time:

Programs $\subset$ $ASCII^*$

(Profs Pigeons)

word string (linguists) ([Sip])

Inputs = $ASCII^*$

(Ice Cream)

Languages = Power$(ASCII^*)$

= Set of all subsets of $ASCII^*$

$\text{Power}(S) = 2^S$

$\stackrel{\text{def}}{=}$ set of all subsets of S

Cantor's Theorem!

$|S|$ is ~~bigger~~ smaller than $|2^S|$

$= |\text{Power}(S)|$

$= |\{\text{set of all subsets of } S\}|$

=

Finite case:

10 profs
pigeons

11 bird sanctuaries
$\underbrace{\hphantom{sanctuaries}}_{\text{languages}}$

P

$f : P \longrightarrow \mathcal{L}$

any map of sets

Idea: If $|P| < |\mathcal{L}|$

then some element of $\mathcal{L}$

is not in the image of f.

What does it mean for infinite set P, $\mathcal{L}$ to say $|P| < |\mathcal{L}|$

strictly less

① $\forall f : P \to \mathcal{L}$

Image(f) is not all of $\mathcal{L}$.

②

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{0, 1, -1, 2, -2, \ldots\}$$

$$\mathbb{N} \underset{\text{strict}}{\subset} \mathbb{Z}$$

fine:

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$

$$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, 3, \ldots\}$$

$$\mathbb{Q} := \{ \text{rational numbers in } \mathbb{R} \}$$

S is countable

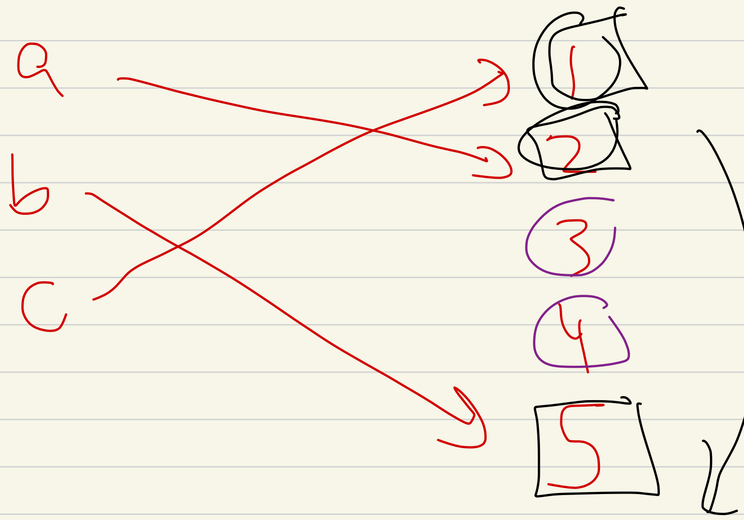
min break

$$f : S \to T$$

(Corrected from $\nexists$ by Andre R. on Sept 21 2021 Office Hours)

$$\text{Image}(f) = \{ t \in T \mid \exists\, s \in S \text{ s.t. } f(s) = t \}$$

Profs $\xrightarrow{f}$ Bird Sctn



$$\text{Image}(f) = \{1, 2, 5\}$$

Not in image

Power

Power$(\{1,2\})$

$$= \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$$

Power$(\{1,2,3\})$

$$= \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$$

$\{1, 2\}$ same $\{2, 1\}$

$(1, 2)$ not same $(2, 1)$

strings over a, b :

$\{ \varepsilon, (a), (b), (a,a),$
$(a,b), (b,a), (b,b), \ldots$

Sequences

typically write

(a, a, b, a)

as aaba

Hi there!

$\downarrow$

(H, i, ␣, t, h, e, r, e, !)

Chapter 0 of [Sip]

Now Cantor's Theorem with 3×3 example

2×2 ?

4×4 ?

Break

Universal TM, Encode, etc ...

A set, S, is countably infinite if it is in bijection with

1. $\mathbb{N} = \{1, 2, 3, \ldots\}$

2. ~~There exists a sequence~~

~~$s_1, s_2, s_3, \ldots, \quad s_i \in S$~~

~~and~~

~~$\{s_1, s_2, \ldots\} = S$~~

Finite, Countably Infinite

Uncountable

Countable = Either Finite

or Countably Infinite

Finite ✓

Countably Infinite

$$\mathbb{N} \xrightarrow{\;g\;} \mathbb{Z}$$

$$1 \longmapsto 0$$

$$2 \longmapsto -1$$

$$3 \longmapsto 1$$

$$4 \longmapsto -2$$

$$5 \longmapsto 2$$

$$\vdots \qquad\qquad \vdots$$

for all $n = \{0, 1, 2, \ldots\}$

$$g(2n+1) = n$$

for $n = 1, 2, \ldots$

$$g(2n) = -n$$

$=$

$\mathbb{Z} =$

$$\{0, -1, 1, -2, 2, \ldots\}$$

exhaustive sequence

Caveat:

$$\boxed{\begin{array}{l} \{1,2\} = \{1,2,2,2,2,\ldots\} \\ \{1,2\} \leftrightarrow s_1, s_2, s_3, s_4, \ldots \\ \qquad\qquad\quad 1, 2, 2, 2, \ldots \end{array}}$$

COUNTABLE

(2) There exists a sequence

$s_1, s_2, s_3, \ldots, \quad s_i \in S$

and

$\{s_1, s_2, \ldots\} = S$

The rationals are countable

$0 \quad -1 \quad 1 \quad 2 \quad -2 \quad 3 \ \ldots$

1	$0/1$	$-1/1$	$1/1$	$\ldots$
2	$0/2$	$-1/2$	$1/2$	$\ldots$
3	$0/3$	$-1/3$	$1/3$	$\ldots$
	$\vdots$			

$\frac{0}{1}, \ \frac{-1}{1}, \ \frac{0}{2}, \ \frac{1}{1}, \ \frac{-1}{2}, \ \frac{0}{3}, \ \ldots$

strict bijection

$\frac{0}{1}, \frac{-1}{1}, \cancel{\frac{0}{2}}, \frac{1}{1}, \frac{-1}{2}, \frac{0}{3}, \ldots$

$0, -1, \cancel{0}, 1, \frac{-1}{2}, \cancel{0}, \ldots$

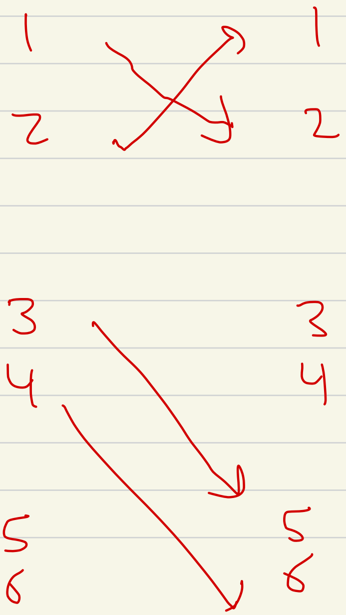
$\mathbb{N} \subset \mathbb{Z}$ but

can be put into

1-1 bijection

All this (and more) in [Sip] § 4.2

Exercise: Show that there are uncountably many bijections $\mathbb{Z} \to \mathbb{Z}$



$$\underbrace{\text{ASCII}^*}_{\text{countable}} \xrightarrow{g} \underbrace{(\text{bijections } \mathbb{Z}\to\mathbb{Z})}_{\text{Uncountable}}$$

$\Longrightarrow$

there is no way to describe all bijections $\mathbb{Z}\to\mathbb{Z}$ in "words"

Thm: If g is a map

$$g: \text{ASCII}^* \longrightarrow (\text{bijections } \mathbb{Z}\to\mathbb{Z})$$

then g is not surjective,

Image(g) is not all

of $\quad$ (bijection $\mathbb{Z} \to \mathbb{Z}$)

$$\begin{array}{ccc} 10 & & 11 \\ \text{Program} & \xrightarrow{f} & \text{Languages} \end{array}$$

Insist by this notation that f is a function

$$\begin{array}{ccc} 1 & \longmapsto & 1 \\ 2 & \longmapsto & 2 \\ \vdots & & \vdots \\ 10 & \longmapsto & 11 \end{array}$$

So f here is a

fixed interpretation.

exhaustive sec

vs

" " with distinct elements