GROUP HOMEWORK 10, CPSC 421/501, FALL 2021

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

THIS HOMEWORK IS NOT TO BE HANDED IN. IT IS QUITE SHORT.

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\mathrm{acc}}, q_{\mathrm{rej}})$ be a one-tape Turing machine. Consider the configuration notation in class on November 30 and in [Sip], Chapter 3. For example: say that a one-tape TM is in state q_2, and contents of the tape is

$$abba \sqcup xabbax \sqcup \sqcup \sqcup \ldots$$

where $\sqcup$ is the blank symbol, and $x, a, b \in \Gamma$; for simplicity we assume that the sets Q and Γ have no intersection (otherwise we have to work with the *disjoint union*[1]. Say that the tape head is on cell number two. Then we denote this configuration by

$$aq_2bba \sqcup xabbax$$

having dropped the contiguous infinite string of $\sqcup$'s that extends infinitely to the right at some point, and having inserted q_2 in the second position to indicate the tape head position. Similarly the strings

$$abbq_2a \sqcup xabbax, \quad abbaq_2 \sqcup xabbax$$

denote the same configuration, except that the tape head location is, respectively, on cells numbers four and five.

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[1]The disjoint union is a limit, and defined only uniquely up to unique isomorphism, and is not defined in any "canonical" way. (The term "canonical" is not precise, but most humans get a feel for what this means.)

(1) Say, with the above understanding, a one-tape TM is in state q_0, and the contents of the tape is

$$abba \sqcup \sqcup \sqcup \ldots.$$

What is the formal representation configuration of the Turing machine when:

(a) the tape head is on cell one?
(b) the tape head is on cell two?
(c) the tape head is on cell four?
(d) the tape head is on cell eight?

(2) Given a one-tape Turing machine, is the set of all possible configurations of the Turing machine countably infinite? Explain.

(3) In what sense can one view a Turing machine transition from one step to another as a "local computation"? [Hint: consider the above two problems, Section 2 of the recently posted handout on the Cook-Levin theorem, class notes, and/or [Sip], Chapter 3.]

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