GROUP HOMEWORK 6, CPSC 421/501, FALL 2021

JOEL FRIEDMAN

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

---

(1) Using the algorithm to convert a regular expression to an NFA, build an NFA the recognizes $L \subset \{a,b\}^*$ described by the expression $(aa \cup abaa \cup aaba)^*$. Briefly explain how you obtained your NFA.

(2) Use the Myhill-Nerode theorem to show that

$$L = \{0^n 1^m \mid n, m \in \mathbb{N},\ n \geq m\}$$

is nonregular. In other words, find an infinite number of words, w, over $\{0,1\}$ such that $\mathrm{AccFut}_L(w)$ are all different.

In all the exercises below, for any $k \in \mathbb{N}$, let C_k be, as usual,

$$C_k = \bigl\{w \in \Sigma^* \mid \textbf{the } k\textbf{-th last symbol of } w \textbf{ is } a\bigr\},$$

where $\Sigma = \{a,b\}$.

(3) (a) Explain why

$$\mathrm{AccFut}_{C_3}(aaa) \neq \mathrm{AccFut}_{C_3}(baa),$$

by finding a word in $\mathrm{AccFut}_{C_3}(aaa)$ that does not lie in $\mathrm{AccFut}_{C_3}(baa)$.

(b) Similarly, show that for any symbols $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \in \Sigma$ we have

$$\mathrm{AccFut}_{C_3}(a\sigma_1\sigma_2) \neq \mathrm{AccFut}_{C_3}(b\sigma_3\sigma_4).$$

Research supported in part by an NSERC grant.

(c) Similarly, show that for any symbols $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \in \Sigma$ we have
$$\mathrm{AccFut}_{C_3}(\sigma_1 a \sigma_2) \neq \mathrm{AccFut}_{C_3}(\sigma_3 b \sigma_4).$$
(d) Similarly, show that for any symbols $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \in \Sigma$ we have
$$\mathrm{AccFut}_{C_3}(\sigma_1 \sigma_2 a) \neq \mathrm{AccFut}_{C_3}(\sigma_3 \sigma_4 b).$$
(e) Using the above, show that if w, w' are distinct strings of length 3 over Σ we have
$$\mathrm{AccFut}_{C_3}(w) \neq \mathrm{AccFut}_{C_3}(w').$$

(4) (a) Find a word, w, of length three over Σ such that
$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(\epsilon).$$
(b) Find a word, w, of length three over Σ such that
$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(a).$$
(c) Find a word, w, of length three over Σ such that
$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(b).$$
(d) If w' is any word over Σ of length two or less, explain how to find a word, w, of length three over Σ such that
$$\mathrm{AccFut}_{C_3}(w) = \mathrm{AccFut}_{C_3}(w').$$

(5) (a) Using the previous two problems, show that there is a DFA recognizing C_3 that has 8 states, where each state is reached from the initial state by following a different word of length 3 over Σ, and that is the minimum number of states.
(b) In this DFA, if you are in the state corresponding to aaa, to which state do you transition upon reading an a? And which upon reading a b?
(c) In this DFA, if you are in the state corresponding to bab, to which state do you transition upon reading an a? And which upon reading a b?

(6) Using the Myhill-Nerode theorem, and the ideas from the previous three problems, show that for any $k \in \mathbb{N}$ there is a DFA with 2^k states recognizing C_k, and that this is the smallest possible number of states.

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA.
E-mail address: jf@cs.ubc.ca
URL: http://www.cs.ubc.ca/~jf