Marks

[10] **1.** Give an explicit description of a Turing machine that takes as input, $x \in \{0,1\}^*$, and (1) accepts x if the first character of x equals the last character, and (2) rejects x if not. You should **explicitly write** your choice of $Q, \Sigma, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$ and **intuitively explain** how the machine works. For example, you should write $\Sigma = \{0,1\}$, since this is the input alphabet.

Answer: The idea is that we read the first character, transition to one state if we see a "0," otherwise transition to another. Then we move right until we see a blank, and then back up one cell and see if match or not.

Specifically we can take $Q = \{q_0, q_{R0}, q_{R1}, q_{L0}, q_{L1}, q_{\text{accept}}, q_{\text{reject}}\}$**,** Γ **to be just** Σ **plus a blank, and let** δ **take the following values below (the values not specified don't matter):**

$\delta(q_0, 0) = (q_{R0}, 0, R)$**,** $\delta(q_0, 1) = (q_{R1}, 1, R)$**,** $\delta(q_{R0}, x) = (q_{R0}, x, R)$ **and** $\delta(q_{R1}, x) = (q_{R1}, x, R)$ **for** $x = 0, 1$**,** $\delta(q_{R0}, b) = (q_{L0}, , L)$ **and** $\delta(q_{R1}, b) = (q_{L1}, , L)$ **where** b **is the blank symbol,** $\delta(q_{L0}, 0) = (q_{\text{accept}}, ,)$**,** $\delta(q_{L1}, 1) = (q_{\text{accept}}, ,)$**,** $\delta(q_{L0}, 1) = (q_{\text{reject}}, ,)$**,** $\delta(q_{L1}, 0) = (q_{\text{reject}}, ,)$**. Also,** $\delta(q_0, b)$ **is your choice of accept or reject.**

Continued on page 3

[10] **2.** Let $\mathcal{P} = \mathcal{I} = \{1, 2, 3, \ldots\}$, the set of positive integers.

(a) Can there be a Result function with the property that every language in $\mathcal{I}$ is accepted by some element of $\mathcal{P}$? Explain.

Answer: We know $|\mathcal{P}| < |2^{\mathcal{P}}| = |2^{\mathcal{I}}|$. Hence no map from $\mathcal{P}$, the set of programs, to $2^{\mathcal{I}}$, the set of languages, can be surjective. Hence there is always some language that is not accepted by any program.

(b) Let Result(p, i) (for $p \in \mathcal{P}$ and $i \in \mathcal{I}$) be defined to be `yes` if $p > i$, `no` if $p < i$, and `loops` if $p = i$. For each $p \in \mathcal{P}$, describe the language that p accepts. Is any p a decider? Describe a language not accepted by any $p \in \mathcal{P}$.

Answer: p accepts the language of integers that are strictly less than p. No p is a decider, since p on input p loops. Any set which is not of the form $\{1, 2, \ldots, p-1\}$ for some p will not be accepted by any program; for example, $\{1, 3\}$, the set of primes, any infinite set, the set of even positive integers less than 25, etc.

Continued on page 4

[10] **3.** In class we showed that $|S| < |2^S|$ for any set S, where 2^S is the set of all subsets of S. We argued that otherwise there is a bijection $f : S \to 2^S$, and then we considered:

$$T = \{s \in S \mid s \notin f(s)\}.$$

How do we obtain a contradiction? Explain.

Answer: Since f is bijective, there is a $t \in S$ such that $f(t) = T$. Now (exactly) one of the following must be true: (1) $t \in T$, or (2) $t \notin T$. If (1) holds, then $t \in T$; but by definition of T, t must satisfy $t \notin f(t)$, which contracts (1). On the other hand, if (2) holds, then $t \notin f(t)$; but by definition of T, this means that t is not among the values of s for which $s \notin f(s)$, and so $t \in f(t)$; but this contradicts (2). So either way we get a contradiction.

Continued on page 5

[10] **4.** Any string over $\{0, 1, A\}$ is uniquely expressible as $n_1 A n_2 A \dots n_k A n_{k+1}$, where $n_1, \dots, n_k$ are strings over $\{0, 1\}$.

(a) Give a high level description of a Turing machine that on input $w \in \{0, 1, A\}^*$, with $w = n_1 A n_2 A \dots A n_{k+1}$, moves the tape head to the n_1-th occurrence of A if it exists, where we view n_1 as an integer in binary notation. Roughly how many extra tape symbols will you need? Show that you can perform this task in time order $|w|^2$.

Answer: There are many ways of doing this. One way (on a 1-tape machine) is to alternate between moving right until you hit an A, then marking it with a new symbol, such as A', and then moving left until you return to n_1 and then decrement n_1 by one. You will probably want to mark the leftmost and rightmost character of n_1 to aid the decrementing procedure, so you may want tape symbols like $1_R, 1_L, 0_R, 0_L$. You could use some of these symbols in two or more different functions, reducing the number of symbols. Each step of moving right and marking an A, then moving left and decrementing n_1 will take $O(|w|)$ steps; since there are at most $|w|$ occurrences of A, the total time is $O(|w|^2)$.

(b) Explain the relevance of an algorithm similar to Part (a) to designing a universal Turing machine, U. [Hint: U's input contains a description of all the values of δ, the transition rule, of a Turing machine to be simulated.]

Answer: The input of U is a Turing machine description, which includes a list of δ values demarcated by some separators. To find the specific δ value we need to apply at each step, we have to move into this δ function description over a certain number of markers. This would require some sort of procedure as in part (a).

Continued on page 6

The End

Be sure that this examination has 6 pages including this cover

The University of British Columbia

Midterm Examinations - October 2011

Computer Science 421/501

Closed book examination | Time: 50 minutes

Name ____________________ **Signature** ____________________

Student Number ____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		10
2		10
3		10
4		10
Total		40