

Marks

- [10] 1. Describe a Turing machine that takes as input,  $x \in \{0, 1\}^*$ , and (1) accepts  $x$  if  $x$  somewhere has two consecutive 1's, and (2) rejects  $x$  if not. You should **explicitly write** and **explain** each of  $Q, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$ .

**Answer:** Let  $\Gamma = \{0, 1, \text{blank}\}$ ,  $Q = \{q_0, q_1, q_2, q_3\}$  with  $q_2 = q_{\text{accept}}$ ,  $q_3 = q_{\text{reject}}$ . Let  $y$  be arbitrary and let  $\delta$  have the values  $\delta(q_0, 0) = (q_0, y, R)$ ,  $\delta(q_0, 1) = (q_1, y, R)$ ,  $\delta(q_0, \text{blank}) = (q_3, y, R)$ ,  $\delta(q_1, 0) = (q_0, y, R)$ ,  $\delta(q_1, 1) = (q_2, y, R)$ ,  $\delta(q_1, \text{blank}) = (q_3, y, R)$ . No need to specify  $\delta$  at the accept and reject states. The state  $q_0$  represents when you have just seen a 0,  $q_1$  when you have just seen one 1 but not two 1's,  $q_2$  when you have just seen two 1's (accept), and  $q_3$  when you have just seen a blank (without being in the accept state) (hence you reject).

[10] 2. Let  $f$  be the functions defined by

$$f(\text{yes}) = \text{yes}, \quad f(\text{no}) = \text{no}, \quad f(\text{loops}) = \text{no}.$$

Consider the axiom: For all  $P \in \mathcal{P}$  there is a  $P' \in \mathcal{P}$  such that for all  $x \in \mathcal{I}$  we have

$$\text{Result}(P', x) = f(\text{Result}(P, x)).$$

Argue in a few sentences either that the axiom is reasonable (i.e., achievable for a Turing machine, C language program, etc.) or unreasonable (i.e., would lead to a contradiction involving what we know from the other axioms).

**Answer: This axiom is unreasonable. If  $L$  is accepted by a program,  $P$ , then this axiom implies that it is decided by a program,  $P'$ . Hence any acceptable language would be decidable, but we know that  $L_{\text{yes}}$  is acceptable but not decidable.**

- [10] 3. Axiom 2 asserts the existence of a “universal program,”  $U \in \mathcal{P}$  such that for each  $P$  and  $x$  we have

$$\text{Result}(U, \text{EncodeBoth}(P, x)) = \text{Result}(P, x).$$

- (a) Why is this called a universal program? Answer in one sentence.

**Answer:**  $U$  simulates any program (given a description of the program).

- (b) Describe in one sentence the relevance of  $U$  to  $L_{\text{yes}}$ ; recall that

$$L_{\text{yes}} = \{\text{EncodeBoth}(P, x) \mid P \in \mathcal{P}, x \in \mathcal{I}, \text{ and } \text{Result}(P, x) = \text{yes}\}.$$

**Answer:**  $U$  accepts  $L_{\text{yes}}$ .

- (c) Describe in one or two paragraphs how you would construct a universal program for your favourite standard programming language (e.g., C, Java, etc.) or a Turing machine.

**Answer:** Let's say for a Turing machine. One could have a multitape Turing machine for the universal machine,  $U$ , with the following functions for each tape: one tape to store the input; one tape to store the simulated machine's work tape (each simulated cell being represented by a number of  $U$  cells, depending on the size of the simulated machine's input), including the tape head position; one tape to store the current state of the simulated machine; one additional tape to manage operations as we simulate one step of computation.

The simulation works by storing the machine configuration on the simulating tape plus the tape that remembers the state. To augment to the next state we look at the tape head of the simulating tape and what is written as the current state. Then we run through each value of  $\delta$ , as specified on the input (we never change the tape cell values on the input tape), see if we match the current state and cell value, and once we reach the matched delta value of the arguments we write appropriately, change states appropriately, and move the simulating tape's tape head left or right as specified.

- [10] 4. Let  $L_1$  and  $L_2$  be languages that are decidable in polynomial time.
- (a) Argue in one or two sentences that  $L_1 \cap L_2$  is decidable in polynomial time.

**Answer:** Run the Turing machine for  $L_1$ , then that for  $L_2$ , and accept the input if both Turing machines accept. The total time is the sum of the times of the two machines, which is polynomial in the input (since the sum of two polynomials is a polynomial).

- (b) Argue in one paragraph that  $L_1^*$  is decidable in polynomial time.

**Answer:** Run dynamic programming on input  $w = w_1 \dots w_n$ , letting  $p(i, j, w)$  be true if  $w_i \dots w_j \in L_1^*$ . We run for all possible  $i, j$ , first over those pairs with  $j = i$ , then with  $j = i + 1$ , then  $j = i + 2$ , until  $j = i + (n - 1)$  (for which there is only  $i = 1, j = n$ ). Each step we use  $p(i, j, w)$  is true iff  $w_i \dots w_j \in L_1$  or for some  $k$  with  $i < k < j$  we have  $p(i, k, w)$  and  $p(k + 1, j, w)$  are true. In very rough terms each such evaluation of a  $p(i, j, w)$  takes at most one query to membership in  $L_1$  plus another order  $j - i$  time, for a total of at most order  $n^2 p(n) + n^3$ , where  $p(n)$  bounds the time to test membership in  $L_1$  for a word of at most  $n$  characters.

**The End**

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The University of British Columbia  
Midterm Examinations - November 2010

Computer Science 421/501

Closed book examination

Time: 50 minutes

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

**Special Instructions:**

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

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**1. Each candidate should be prepared to produce his library/AMS card upon request.**

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No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

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(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

**3. Smoking is not permitted during examinations.**

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Total		40