Marks

[5] **1.** Write a DFA describing the language of strings over $\{0, 1\}$ that have an even number of 1's. Use the technique discussed in class and the book to use the DFA to obtain a regular expression for this language (by writing the DFA as a GNFA and then repeatedly reduce the number of states in the GNFA).

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[5] **2.** Use the pumping lemma for context-free languages to show that the language $L = \{0^n 1^n 2^n \mid n = 0, 1, 2, \ldots\}$ is not context-free.

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[5] **3.** Let $L = \{a^{100}\}$. Argue that a DFA that recognizes L must have at least 101 states. Explain your argument from scratch; i.e., if you use want to use Myhill-Nerode, then explain why it is true in this case.

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[5] **4.** Describe a Turing machine that takes as input, $x \in \{a, b\}^*$, and (1) accepts x if $|x|$ is even, and (2) rejects x if $|x|$ is odd. You should **explicitly write** and **explain** each of $Q, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}, \delta$.

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[5] **5.** In the following questions you may assume that SAT is NP-complete. Let DOUBLE-SAT be the set of $\langle \phi \rangle$ such that ϕ is a Boolean formula with at least two satisfying assignments. Show that DOUBLE-SAT is NP-complete.

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[5] **6.** Consider the following "Funny Axiom": given any program, P, there is a program, P', such that

$$\text{Result}(P', x) = f(\text{Result}(P, x)),$$

where $f(\texttt{yes}) = \texttt{loops}$ and $f(\texttt{loops}) = f(\texttt{no}) = \texttt{yes}$. Show that if we add the Funny Axiom to all the axioms in the handout we get an inconsistency.

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[5] **7.** Give short explanations to the following questions.

(a) Show that SAT $\leq_P L_{yes}$.

(b) Explain why part (a) does not imply that L_{yes} is NP-complete.

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[5] **8.** Outline (in a few sentences) the reduction 3SAT $\leq_{\mathrm{P}}$ SUBSET $-$ SUM done in class and the text, and illustrate the reduction on the example $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3)$.

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The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Final Examinations - December 2009

Computer Science 421/501

Closed book examination Time: 150 minutes

Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

Calculators, notes, or other aids may not be used. Answer questions on the exam. This exam is two-sided!

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

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