CPSC 421/501 Oct 29

Chapter 7:

- Big-O, the classes TIME($t(n)$), $\underbrace{\text{NTIME}(t(n))}$

(non-deterministic)

we use Def 7.21, not Def 7.18, although they are equivalent

- P and NP

- Reductions, Poly time functions and NP-completeness

- Start on Cook-Levin Theorem

Breakout Room Questions:

① Show that

$$SAT = \left\{ \langle f \rangle \;\middle|\; \begin{array}{l} f \text{ is a satisfiable} \\ \text{Boolean formula, i.e.} \\ \text{for some } x_1, \dots, x_n \in \{T, F\} \\ f(x_1, \dots, x_n) = T \end{array} \right\}$$

is in NP

② Show that

$$3COLOUR = \left\{ \langle G \rangle \;\middle|\; \begin{array}{l} G \text{ is a graph that can} \\ \text{be 3 coloured, i.e.} \\ \exists \text{ map } V \to \{1,2,3\} \text{ s.t.} \\ \text{no edge is monochromatic} \end{array} \right\}$$

is in NP

(3) Show that

integers — target integers

$$\text{SUBSET-SUM} = \left\{ \langle x_1, \dots, x_k, t \rangle \;\middle|\; \begin{array}{l} \text{s.t. } x_1, \dots, x_k, t \in \mathbb{N} \text{ and} \\ \text{for some } I \subset \{1, \dots, k\} \\ \sum_{i \in I} x_i = t \end{array} \right\}$$

is in NP-complete

(4) If $f : \Sigma_1^* \to \Sigma_2^*$ is poly time computable, and $g : \Sigma_2^* \to \Sigma_3^*$ is as well, is

$$g \circ f : \Sigma_1^* \to \Sigma_3^*$$

also poly-time computable?

ADMIN: MIDTERM

- There is a gradescope assignment (not for credit) "Make sure you midterm submission is legible," due "tomorrow night" (late submissions until Nov 2)
- Exam will be 1 hour long, plus 5 minutes to upload your solutions; start 9:30am
- Exam will be delivered via Canvas
- Questions posted under homework section yesterday will be discussed ~~on~~ Tuesday; bring any other questions you have.
  Tuesday = midterm review
- Review homework, try Midterm 2019
- If you have a tablet, you can submitt from there

ADMIN: CPSC 501 presentation!

- There is a Canvas "discussion" for 501 students, regarding presentation days.

- One student had nice additional topic
  - Pumping Lemma (will not be on final)

    vs. Myhill-Nerode

START Ch. 7:

Chapter 7:

(non-deterministic)

- Big-O, the classes TIME(t(n)), NTIME(t(n))

  we use Def 7.21, not Def 7.18, although they are equivalent

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TIME(t(n)) $\quad t(n) = t : \mathbb{N} \to \mathbb{R}_{>0}$

TIME(t(n)) = { L that can be recognized by a TM in time O(t(n)) }

Think of $TIME(n)$ = "linear time solvable problems"

$TIME(n^k)$ = problems solvable in time order n^k

MAKE PRECISE!

If M is a Turing machine, we say M takes time at most $f(n)$, where $f(n) : \mathbb{N} \to \mathbb{R}_{>0}$, if on any input $w \in \Sigma^*$, M halts (in q_{acc} or q_{rej}) in time $\leq f(|w|)$,

(time = # of steps of M).

Say, if f, t, functions $\mathbb{N} \to \mathbb{R}_{>0}$,

$f(n)$ is <u>order</u> $t(n)$, write

$f(n) = O(t(n))$ if there is

n_0, C s.t.

$$f(n) \leq C\, t(n) \text{ for } n \geq n_0$$

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e.g. $n^2 + n\log n + 1 = O(n^2)$

$n^2 + n^2 \log n + 1 = O(n^2 \log n)$

$n^2 + n^2 \log n + 1 = O(n^5)$

$TIME(t(n)) = \{ L$ that are recognized

in time $f(n)$ for some T.M. (that

can be multitape), where $f(n) = O(t(n))\}$

$$\underbrace{P}_{\text{polynomial (time languages)}} = TIME(n) \cup TIME(n^2) \cup TIME(n^3) \cup \ldots$$

$$= \bigcup_{k \in \mathbb{N}} \mathrm{TIME}(n^k)$$

If L can be recognized in time

$$10^{10^{10}} \cdot n^2 + 5$$

then $L \in \mathrm{TIME}(n^2) \subset P$.

Remark: $10^{10^{10}} n^2 + 5 \in O(n^2)$

Notation: $\ddot{O}O(n^2)$ = Uh-oh of n^2

Udi Manbar means
(joke) n^2 times $10^{10^{10^{10}}}$

Pointing out that alg runs in time $O(n^2)$, doesn't mean that it is necessarily realistic to run.

But... class P seems to include many "efficient" algorithms, so P is a reasonable place to start.

Technically $O(n^2) = \{\text{functions}\}$

Now: Introduce Problems:

SAT, SUBSET-SUM, 3COLOR, ...

for which we don't have poly time algorithms... but are all "as difficult"...

$\text{SAT} = \{ \langle f \rangle \mid$ f is Boolean formula that is satisfiable,

$f = f(x_1, \ldots, x_n)$ and

for some $x_1, \ldots, x_n \in \{T, F\}$

$f(x_1, \ldots, x_n) = T \}$

$f = x_1 \wedge (\neg x_2)$ is satisfiable

$= x_1$ and (negation x_2)

$x_1 \to T$ $\quad$ T and (negation F)

$x_2 \to F$

T and T = T

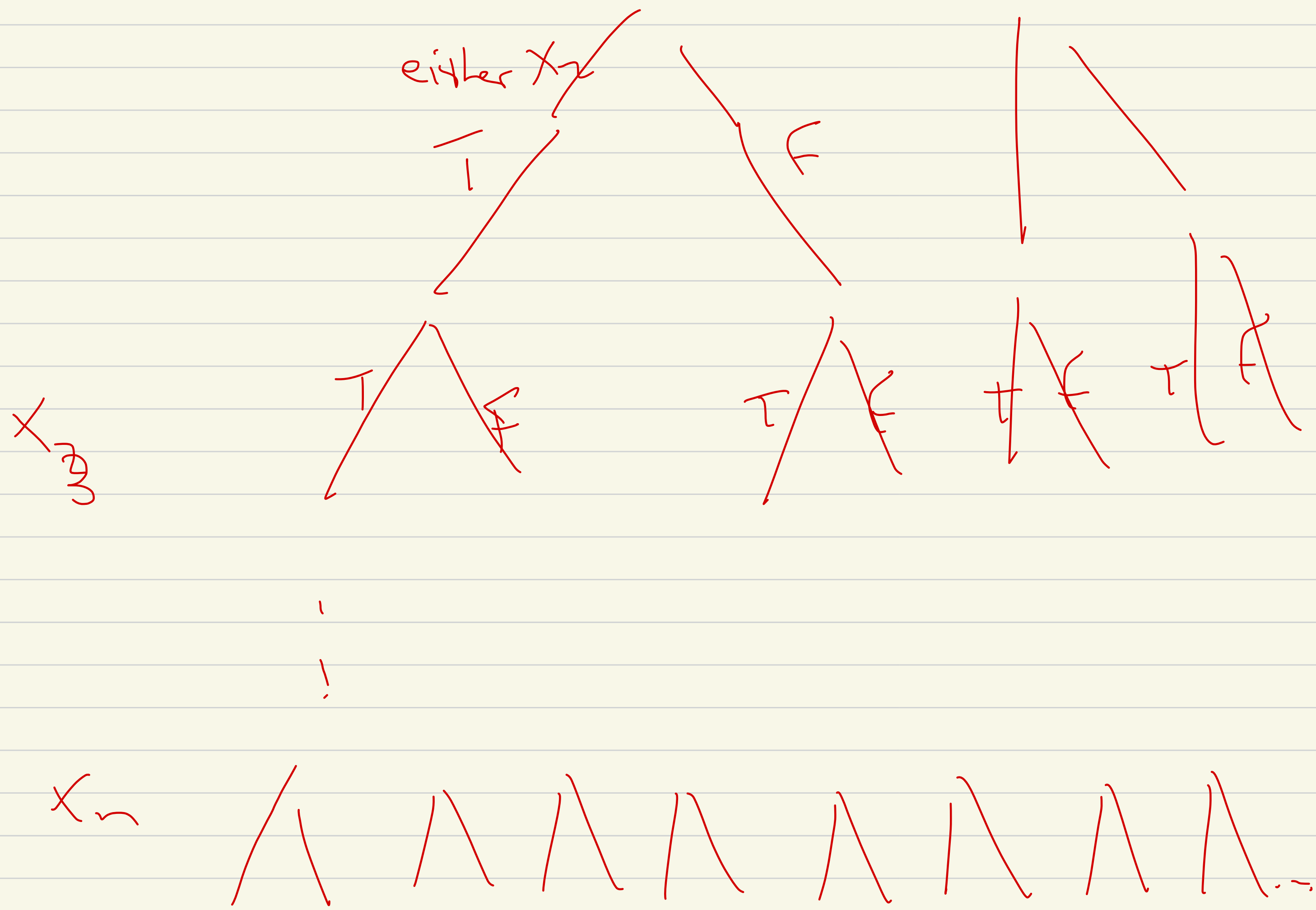
$g = x_1 \wedge (\neg x_1)$ is not satisfiable

$g(T) = T$ and $(\neg T) = T$ and $F = F$

$g(F) = F$

Solve SAT given $f(x_1, \dots, x_n)$

either $x_1 = T$ or F



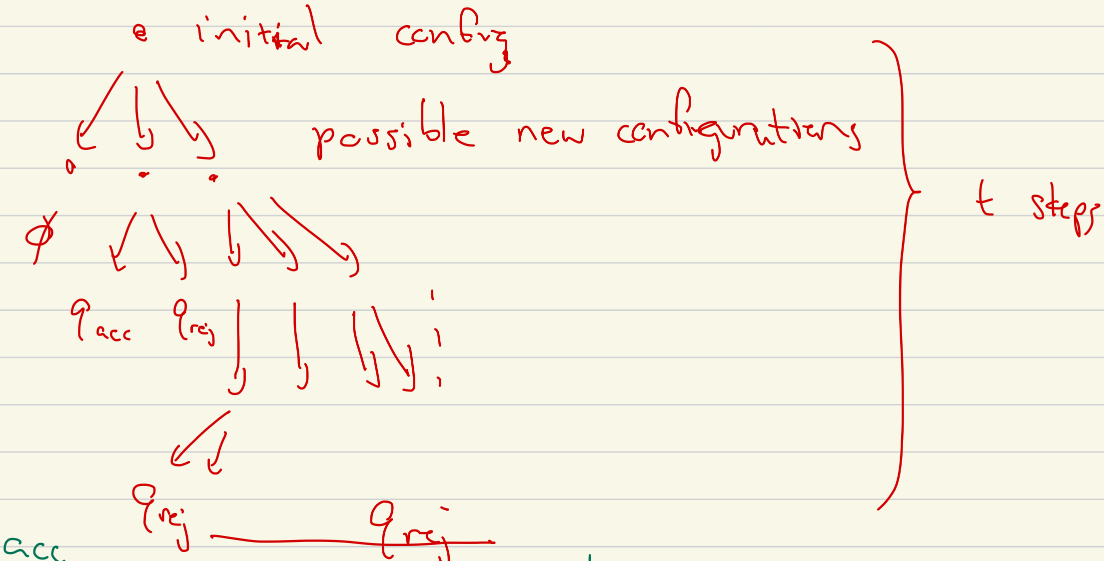
f is satisfiable iff one leaf evaluates to T

We have DFA vs NFA   N = non-determinism

TM vs $\underbrace{\text{non-deterministic TM}}$

what does "time" mean w/ NP

If M is a non-deterministic TM



"decide"

if there is at least one q_{acc} comp path, then the input is accepted

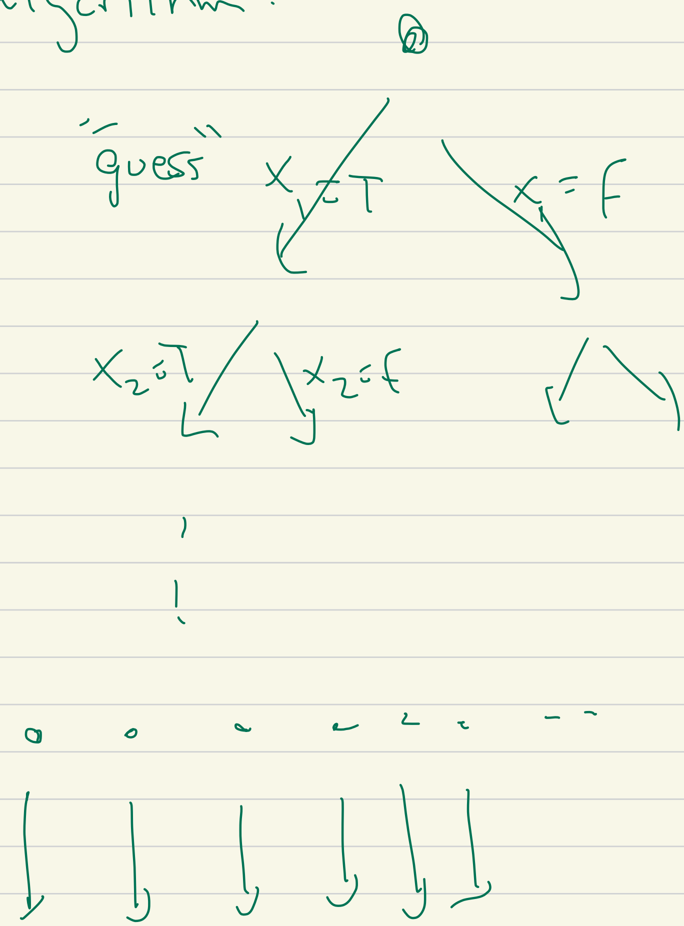
Say that M runs in time t if every possible configurations stops in time $\leq t$.

We say M runs in time $f(n)$ $(f: \mathbb{N} \to \mathbb{R}_{>0})$ if on any input $w \in \Sigma^*$, M runs within time $f(|w|)$

$\text{NTIME}(t(n)) = \{ L$ which are decidable in time $O(t(n))$ by some non-deterministic multi-tape TM $\}$

$$NP = \bigcup_{k \in \mathbb{N}} NTIME(n^k)$$

SAT: algorithm:



is $f = T$?

← all possible 2^n settings of T/F

q_{acc} ← ≥ 1 accept, then f is accepted

Breakout: do ② and ③

$$UNSAT = \left\{ \langle f \rangle \;\middle|\; \begin{array}{c} f \text{ Boolean formulas} \\ \text{not satisfiable} \end{array} \right\} \overset{?}{\in} NP$$

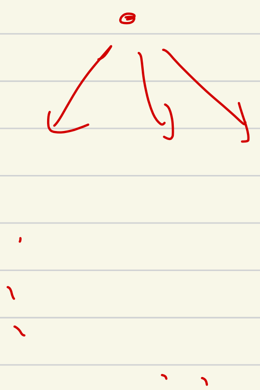
Ultimately: Is $P = NP$?

(8 min breakout)

10:34 – 10:42 Breakout

Remark — UNSAT $\in$ NP ???

Non-det TM



accept input w if at least one path ends in q_{accept}, otherwise reject w.

3COLOUR in NP ?

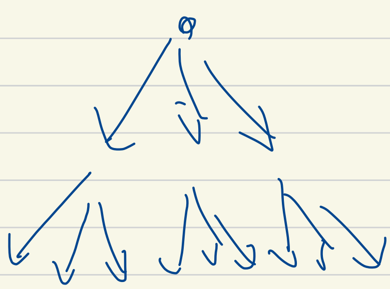
Graph $G = (V, E)$

3-colouring of G : $V \to \{\text{red}, \text{green}, \text{blue}\}$

s.t. each edge has endpoints with distinct colours.

Idea # functions : $V \to \{3 \text{ elt set}\}$

$3^{|V|}$.



"guess" colour of first vertex

" " " 2nd "

⋮

← $3^{|V|}$ configurations

at least one valid 3-coloring, then $G \in 3COLOR$

But UN-SAT

UN-3COLOUR

⋮

If $L \in P$

If $L \in TIME(t(n))$

$\Rightarrow L^{comp} \in TIME(t(n))$

But $L \in NTIME(t(n)) \overset{??}{\Longrightarrow}$ unclear

if $L^{comp} \in NTIME(t(n))$

SAT

poss configs

q_{rej} q_{acc} ← if one q_{acc}, accept f f!(r } }

q_{acc} q_{rej} ← recognize

$\{ \langle f \rangle \mid$ at least one assignment makes $f = F \}$

$\neq \{ \langle f \rangle \mid$ all assignments make $f = F \}$

UNSAT

Class ends

SAT alg,

f has n-variables !

2^n comp path

but each path is poly time.