CPSC 421/501 Oct 22, 2020

- $\langle M \rangle$ description of a "standard" Turing machine
- $\langle M, w \rangle$ " " " " "

  plus an input to M

- There are unrecognizable languages

Ch. 4:

- $A_{TM} = \{ \langle M, w \rangle \mid M$ is a Turing machine that accepts $w \}$

  is undecidable

- A_{TM} is recognizable (by a universal Turing machine)

- Complement(A_{TM}) is not recognizable

Breakout Room Problems

① Give a more detailed description of a universal TM, i.e. that given $\langle M, w\rangle$ can "simulate" M's computation on input w.

② Given that A_{TM} is undecidable, show that $HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halts on input } w\}$ is undecidable

(halt means reaches either q_{accept} or q_{reject}).

③ If L is undecidable and recognizable show that $L^{comp} = \Sigma^* \setminus L$ is unrecognizable

When you "run" an algorithm on a $\begin{cases} \text{graph} \\ \text{DFA} \\ \text{Boolean} \\ \text{Turing machine} \\ \vdots \end{cases}$

$\{\text{graphs}\}$ is vastly uncountable,

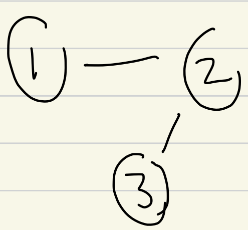
since $V = \{\text{vertex set}\}$ is <u>any</u> <u>finite set</u>

But in practice ...

if $V = \{1, \ldots, n\}$ for some n,

"standardized graph"

then describe graph as a finite string, e.g.



$\rightarrow$ "3 vertices, edges $\{1,2\}, \{2,3\}$"

$\rightarrow$ 3 # 1 # 2 # 2 # 3

$\in \{0, 1, \ldots, 9, \#\}^*$

$\{0, 1, \ldots, 9, \#\}^*$ is countable $\Rightarrow$ #(standard graphs) countable

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Ch 4: Need to <u>standard</u> Turing machines

$$TM \stackrel{\text{def}}{=} (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

But in practice . . .

$Q = \{1, \ldots, q\}, \quad \Sigma = \{1, \ldots, s\}, \quad \Gamma = \{1, \ldots, g\}$

Say $q_0 = 1$, $q_{acc} = 2$, $q_{rej} = 3$

$\delta : \underbrace{\{1, \ldots, q\}}_{[q]} \times \underbrace{\{1, \ldots, g\}}_{[g]} \to [q] \times [g] \times \{L, R\}$

Any TM is <u>equivalent</u> to a standardized TM (by renaming). Then if M is " ",

$\langle M \rangle =$ give q # give g # $\underbrace{\delta(1,1)}_{\delta_Q(1,1) \# \delta_G(1,1) \# {L \atop R}}$ # $\delta(1,2)$ # ... # $\delta(1,g)$ # $\delta(2,1)$ # $\delta(2,2)$ # ... # $\delta(q,g)$

$\langle M \rangle =$ "description of M"

35 # 3 # 7 # $\underbrace{3 \# 6 \# L}_{\delta(1,1)}$ # ... $\underbrace{\# 2 \# 5 \# L}_{\delta(q,g)}$

$\Gamma = \{1, \ldots, g\}$

$\Sigma = \{1, \ldots, s\}$, $\sqcup = s+1$

$\langle M \rangle = \underbrace{\{0, 1, \ldots, 9, \#, L, R\}^*}$

$\{\text{standard TM}\} \subset$ ← is countable

$\{\text{languages over any alphabet}\}$ is uncountable

① If a TM, M, "recognizes" $\{w \in \Sigma^* \mid M \text{ accepts } w\}$

then there are languages (over any alphabet) that are not recognized by any TM.

② (Ch 4 main point:) We will give languages that are not recognizable. (will involve a "self-referencing + negation" arguments)

Recall, a TM machine, M, recognizes: $\{w \in \Sigma^* \mid M \text{ accepts } w\}$

M is a <u>decider</u> if on every input w, M <u>halts</u>, i.e. M eventually reaches q_{accept} or q_{reject}.

<u>Theorem</u>: $Accept_{TM} = A_{TM} = \{\langle M, w\rangle \text{ s.t. } M \text{ accepts } w\}$

first describe M

$w \in \{1, \ldots, 5\}^*$

is not <u>decidable</u>, i.e. there is no TM, H, s.t. H is a decider and H recognizes A_{TM}.

[More famous: $HALT_{TM} = \{\langle M, w\rangle \mid M \text{ halts on } w\}$ is also undecidable.]

Pf: Say that H is a TM that decides A_{TM}. We'll get a contradiction.

Using H, form a TM, D, s.t.

(1) if w, input to D, is of the ~~form~~ form $\langle M \rangle$, then write down $\langle M, \langle M \rangle \rangle$

(2) run H on $\langle M, \langle M \rangle \rangle$, and

D accepts $\langle M \rangle$ if H rejects $\langle M, \langle M \rangle \rangle$

D rejects " " " accepts "

[Rem: if w input to D is not $\langle M \rangle$, then don't care...]

$$\langle M \rangle = 32\#3\#7\#\ldots \#5\#6\#L$$

take on

$$\langle M, \langle M \rangle \rangle =$$

$$\Sigma = \{0, 1, \ldots, 9, \#, L, R\}$$

Now: What happens when D input $\langle D \rangle$?

D on input $\langle D \rangle$:

accept if D on input $\langle D \rangle$ rejects

reject " " " " " accepts

But either D accepts $\langle D \rangle$ $\Longrightarrow$ D rejects $\langle D \rangle$

D rejects $\langle D \rangle$ $\Longrightarrow$ D accepts $\langle D \rangle$

By design D feeds $\langle D, \langle D \rangle \rangle$ to H and does opposite

H accepts $\langle M, \langle M \rangle \rangle$ iff M accepts $\langle M \rangle$

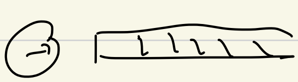
H rejects $\langle M, \langle M \rangle \rangle$ iff M rejects $\langle M \rangle$

D accepts $\langle M \rangle$ $\longrightarrow$ iff M rejects $\langle M \rangle$

" accepts $\langle D \rangle$ $\longrightarrow$ iff D rejects $\langle D \rangle$

This argument shows A_{TM} is undecidable.

Fact 1: A_{TM} is recognizable:

TM: $\langle$  $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$, input $\rangle$: "You can "simulate" M on input w"

Build U s.t. on input $\langle M, w \rangle$:

U accepts if M accepts w

U reject " " reject "

U doesn't halt " " doesn't halt on input w

In [Sip], Theorem 4.1, proof, there is a claim that U exists w/o any details.

Breakout Room Problems 1 & 2

10:29 ~ 10:39

You choose breakout rooms, but <u>Rooms</u> <u>1–6</u> are for people who want "random rooms"

Universal Turing machine: Build a TM

(use multiple tapes)

$\langle M, w \rangle$

$Q, \Sigma, \Gamma, \ldots$

string

32#3#... #R

element of Σ^*

e.g. 3221213

3221213

1-tape

tape 1

input

| 32# ... #R ##3221213 ⊔⊔⊔...

double separator

copy

| 32# ... #R ⊔⊔⊔⊔ stores M, tape 2

universal TM:

| 3221213 ⊔⊔ tape 3

| 1 tape 4

simulated tape

simulated state (here 1,...,32)

simulation

each step of M on w:

describes M — # 1 1 # 6 # R — tape 2

$\delta(q_i, \gamma)$

compute where in M's description — tape 5, 6

class ended

Midterm covers up to Ch 3,

includes descriptions of
- graphs
- T.M
- Boolean formulas

and countably many "standard" TM, graphs, ...