§ 3.2 - **Multitape vs. Single Tape**

Non-deterministic (more important for NP) (Chapter 7)

\[ \delta : Q \times \Gamma \rightarrow \text{Power}(Q \times \Gamma \times \{L, R\}) \]

§ 3.3 - **Descriptions of**
- Graphs
- Boolean formulas
- etc.

Chapter 4: **Goal**: Find problems that are undecidable and unrecognizable

§ 4.1: **Decidable Problems** versus **Recognizable Problems** (examples)

§ 4.2: **Undecidable Problems**
- Universal Turing Machines can recognize
  \( \text{A}_{\text{Tm}}, \text{HALT}_{\text{Tm}} \)
- But \( \text{A}_{\text{Tm}}, \text{HALT}_{\text{Tm}} \) are undecidable.
For now we'll skip of non-determinism, but the idea is like NFA

### DFA

- $q_0$
- $\delta : Q \times \Sigma \to \text{Power}(Q)$

### NFA

- $q_0$
- $\{ 1, 2 \}$
- $\{ 3 \}$
- $\{ 4 \}$

### Non-deterministic TM

- $\delta : Q \times \Gamma \to \text{Power} \left( Q \times \Gamma \times \{ L, R \} \right)$

- $q_0$, string on tape
- $\Sigma$, tape symbol

- $\{ 0 \}$ is a possible value of $\delta$

- Non-determinism important for NP (versus P)
Last time: Multi-tape machines.

\[ \delta : Q \times \Gamma^3 \rightarrow Q \times \Gamma^3 \times \{L, R, S\}^3 \]

E.g. Recognize \( \{ \text{On}^n \mid n = 3, 7, 13, \ldots \} \)

takes roughly quadratic time on 1-tape

linear time on 2-tape machine
Earlier!  

To copy tape 1 to tape 2

First step

A' new symbol to say "A was here" and this is the left edge of tape 1.
Rem: \( \Gamma = \text{symbols written to each tape cell} \)

is an alphabet, so

a finite set

\[
\Gamma = \Sigma \cup \{ \text{u} \} \cup \{ 0, 1, 2, \ldots, 15 \}
\]

\[
\Gamma \\
\]

\[
\begin{aligned}
\text{e.g., TIMES} &= \{ a \# b \# c \mid a, b, c \in \{ 0, 1 \} \\
&\quad \text{st. } \text{binary}(a) \times \text{binary}(b) \\
&\quad = \text{binary}(c) \}
\end{aligned}
\]

convenient to have

a number of tapes
Theorem: If there is a multi-tape Tm recognizes $L$, then there is a 1-tape machine that recognizes $L$.

Ans: Yes (if you don't mind trying more time).

Idea:

1. Put tape 2 at the end of type 

   change to type 2

      maybe b

      read this too

   or cells to tell you where tape heads are

   extra symbols

   type 1
1-type → super big cell

Could be telling you (1) what is \( Y_1 \) on tape 1 at that position

\[ x \times \neg x \times \{ \text{yes: is type } 1 \} \quad \quad \{ \text{no: head 1 here} \}\]

(2) is type head 1 at cell 1?

\[ \neg x \times \text{second } \times \{ \text{yes: } \neg \text{ no} \} \]

(3) what is \( Y_2 \) at type 2?

Turns out: any 2-type machine algorithm that takes time = \( \text{time}(w) \) \( w \) input, you may need \( (\text{time}(w))^2 \) or < 1-type but: 3-type alg, still \( (\text{time}(w))^2 \) on a 1-type
Real issue

\[ 1 - 1, 1 \]

\[ \text{farthest \ can \ move} \]

\[ \text{steps taken} \]

\[ \text{so far} \]

need to "run

than the whole tape contents"=

**time** a TM takes on input \( w = \text{def} \# \text{steps} \) until

\( \text{acc} \ or \ \text{rej} \)

**Space** a TM takes on input \( w = \text{def} \) farthest cell

position (to

the right) that

a tape head

reaches

\[ \text{cell 1} \]

\[ \text{after 10 steps} \]

\[ \text{cell 11} \]

\[ \text{space} \leq \text{time} + 1 \]
So 3-tape initially at cell 1, tells you what is going on at cell 1 of types 1,2,3

if \( \Gamma \) symbols a multitape

Super Cell \( \Rightarrow \) Super big cell

\[ \text{Super Cell} \Rightarrow \text{Super big cell} \]

Break 5 min! 10:24 → 10:29
Finish Ch 3:

§ 3.3: Only talk about "descriptions"

Say you have an algorithm to see
- if a graph is connected
- "..." 3-colourable
- "..." has a clique of size S
- etc.

Technically, graph $G = (V, E)$, here $V$ is a finite set, $E \subseteq$ unordered pairs of $V$

E.g.,

\[ a \rightarrow b \]

\[ \rightarrow c \rightarrow d \]
\[ V = \{ a, b, c, \emptyset \} \]

\[ E = \{ \{a, b\}, \{a, \emptyset\}, \{c, c\} \} \]

(directed graph: \( E \subseteq V \times V \))

Question: \{ all graphs \} countable?

Technically: \( V \) could be a finite set of \( \mathbb{R} \), or of some uncountable set

But, up to renaming the vertex set
Technically:

\[ \left\{ \begin{array}{l} \text{set of graphs} \text{ } \text{uncountable} \\ \text{set of graphs sat.} \\ \{ V = \{1, \ldots, n\} \text{ for some n} \} \end{array} \right\} \]

"Standard graph" is countable

When you tell a computer about a graph,
you can say

\[
\begin{align*}
\text{20 vertices, edges: } \{1,2\}, \{5,8\}, \ldots \{3,12\}
\end{align*}
\]

as a string

\[
2 e \# 1 \# 2 \# 5 \# 8 \ldots \# 3 \# 12
\]

one edge another edge

\[
\epsilon \{0,1,\ldots,9\} \#
\]

So we speak of

\[
\langle G \rangle \text{ as describing a } \langle \text{Standard graph} \rangle \text{ for each finite graph.}
\]
Similarly: Boolean formula:
\[ f = (x_1 \land x_2) \lor \neg x_3 \]

\[ \langle f \rangle = (x_1 \land x_2) \lor \neg x_3 \]

"Standardized Boolean formula" \[ x \cdot 35 \]

\[ \in \{ ( , , \land , \lor , 0 , \ldots , 9 ) \} \]

1. Is the set of TM countable?
2. Can you "standardize TM" to make them countable?
After class

1. Tape

- For "high level" it is
  OK to just say this
  is your first step
- For "formal description"
  you have to give 5

Ready to start