CPSC 421/501 Oct 15, 2020

§3.1 — recognize versus decide, "review" some aspects of TM

§3.2 — k-tape Turing Machines (convenience)

$$\delta : Q \times \Gamma^k \to Q \times \Gamma^k \times \{L,R,S\}^k$$

- Non-deterministic (more important for NP) (Chapter 7)

$$\delta : Q \times \Gamma \to \mathrm{Power}(Q \times \Gamma \times \{L,R\})$$

§3.3 - Descriptions of
  - Graphs
  - Boolean formulas
  - etc.

Chapter 4:

§4.1 : Decidable Problems (examples)

§4.2 : Undecidable Problems

- Universal Turing Machines can recognize

A_{TM}, $HALT_{TM}$

- But A_{TM}, $HALT_{TM}$ are undecidable.

Breakout Room Problems :

high-level
middle : implement – { how many tapes, ...
formal – describe δ

① Give high-level or implementation level of Turing machine to decide :

$$\text{PRIMES} = \{2,3,5,7,11,13,17,\ldots\} \subset \{0,1,\ldots,9\}^*$$

$$\text{TIMES} = \left\{ a\#b\#c \;\middle|\; \begin{array}{l} a,b,c \in \{0,1\}^*, \\ a \cdot b = c \text{ as} \\ \text{base 2 numbers} \end{array} \right\}$$

$\Sigma = \{0,1,\#\}$

Don't do right now

$$\text{3COLOR} = \left\{ \langle G \rangle \;\middle|\; \begin{array}{l} G \text{ is a graph} \\ \text{that can be 3-colored} \end{array} \right\}$$

② Give an algorithm (deterministic Turing machine) to recognize

$$\left\{ \langle p \rangle \;\middle|\; \begin{array}{l} p = p(x,y,z) \text{ is a polynomial over} \\ \text{the integers such that } p(a,b,c) = 0 \\ \text{for some } a,b,c \in \mathbb{N} \end{array} \right\}$$

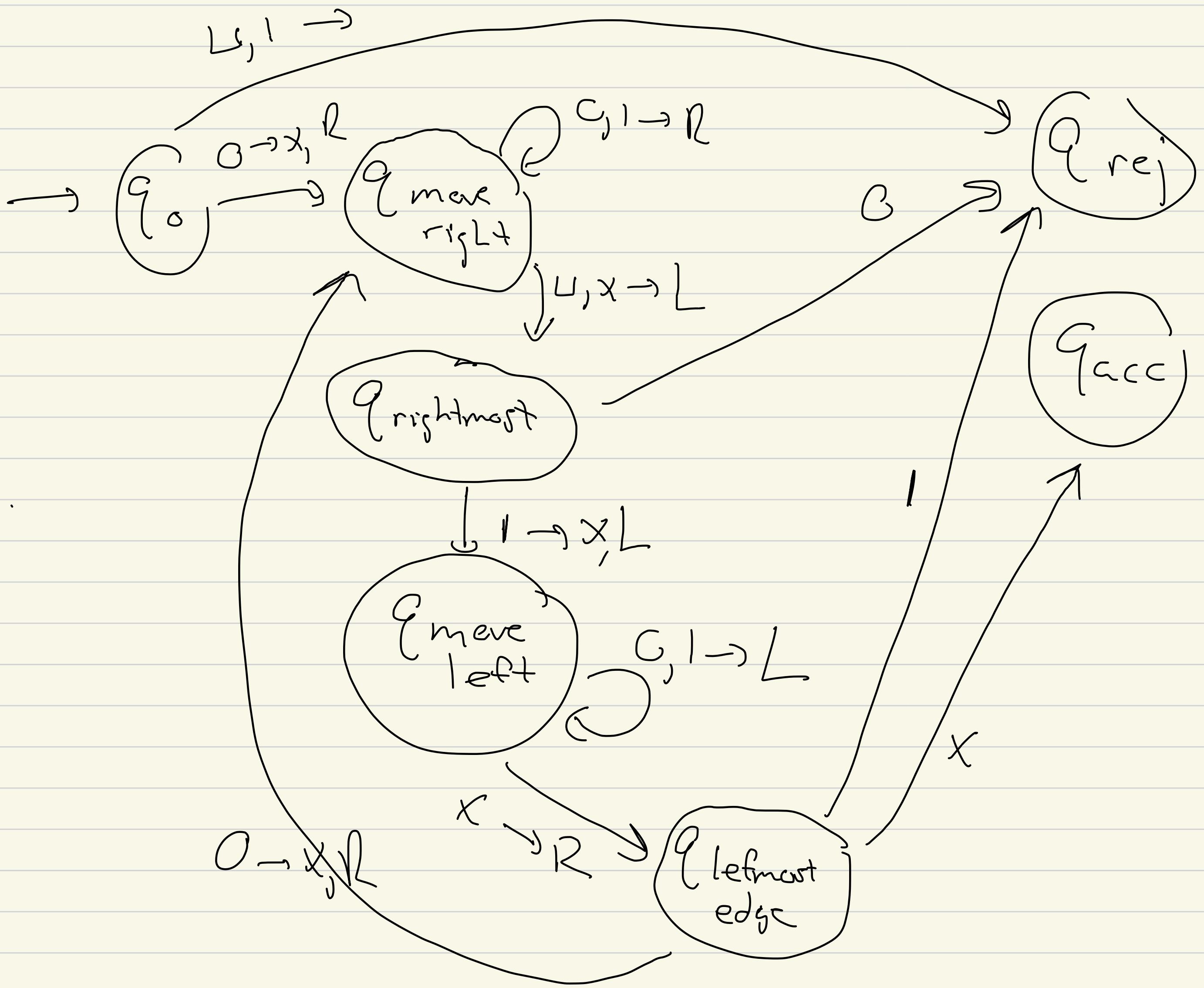
③ What is a reasonable way to describe (over some finite alphabet):

- a Boolean formula ?
- a polynomial $p(x,y)$ of x,y with integer coefficients ?
- a DFA ?
- a Turing machine ?

④ Is the set of Turing machines countable ?

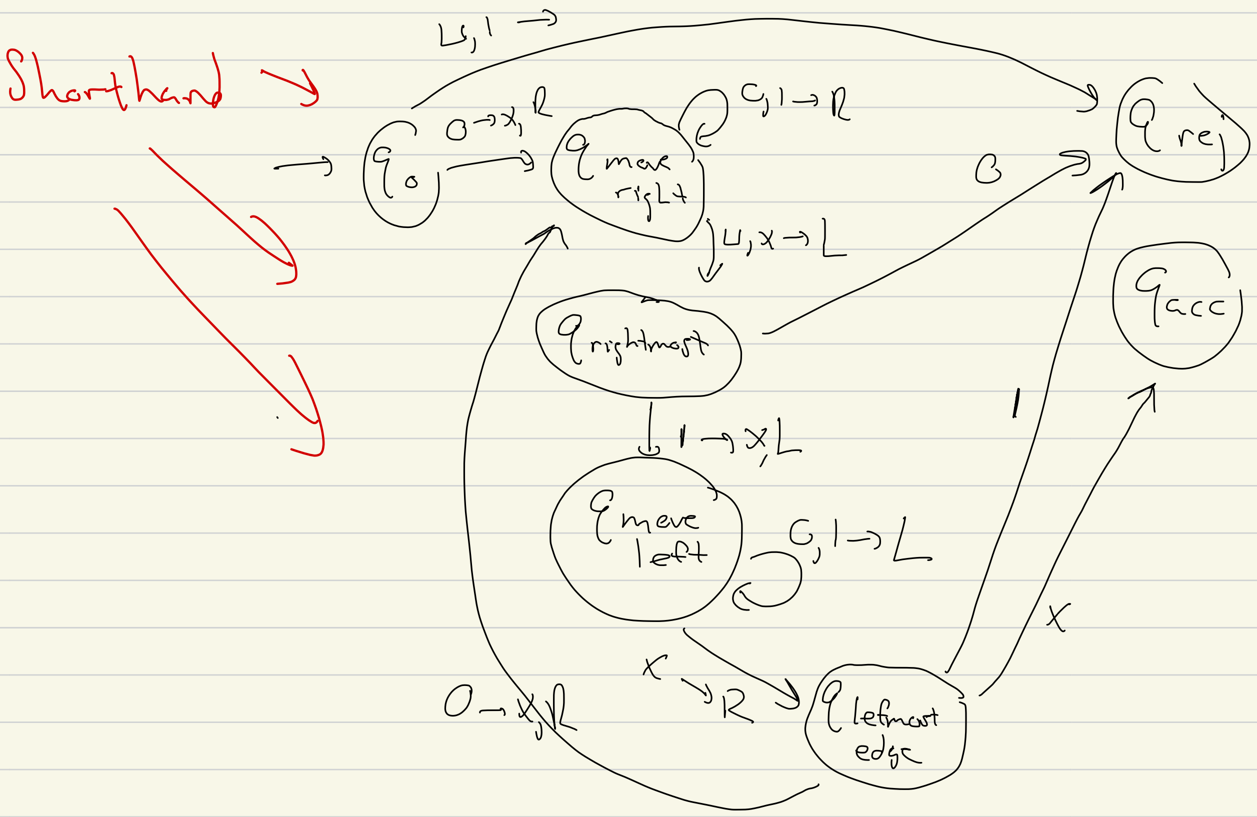
⑤ Is the set of "Turing machines algorithms" (where you identify two machines that "run the same algorithm") countable?

Last time: For $\{0^n1^n \mid n=1,2,\dots\}$ we built:



Question: Given a string as input for a given TM (Turing machine) — (M, input) — can we figure out if M accepts the input?

Ans: There is no algorithm that stops in a finite amount of time and tells you the answer...



Check on input $01 \in L = \{0^n1^n \mid n \in \mathbb{N}\}$

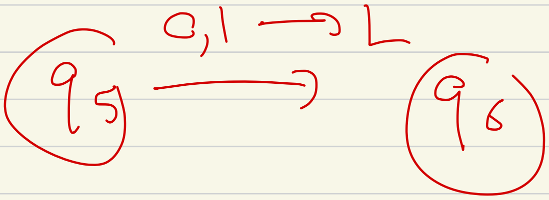
Formalities: TM is $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

$\delta: Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$

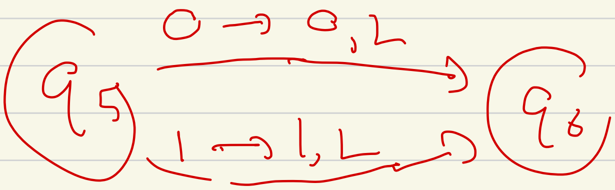
new state

before move tape head, write this symbol

Shorthand:

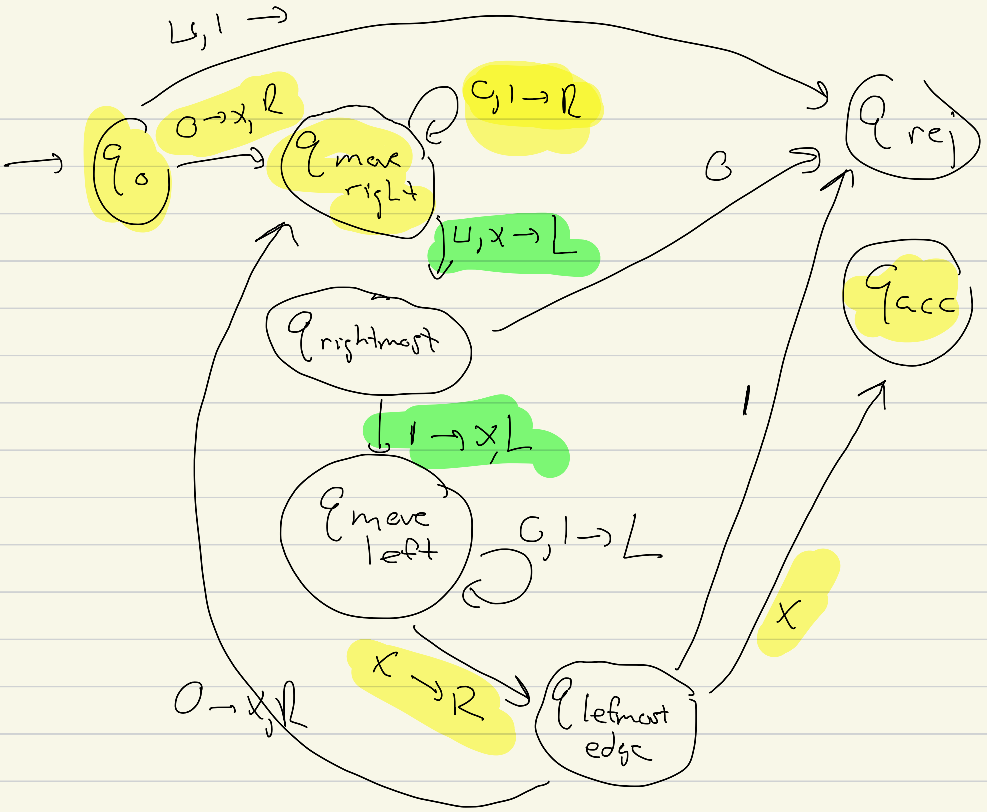


means



$\delta(q_5, 0) = (q_6, 0, L)$

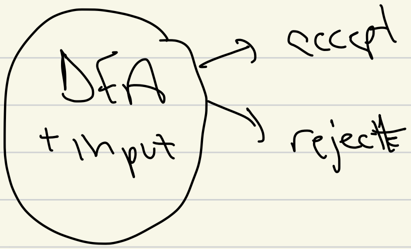
$\delta(q_5, 1) = (q_6, 1, L)$

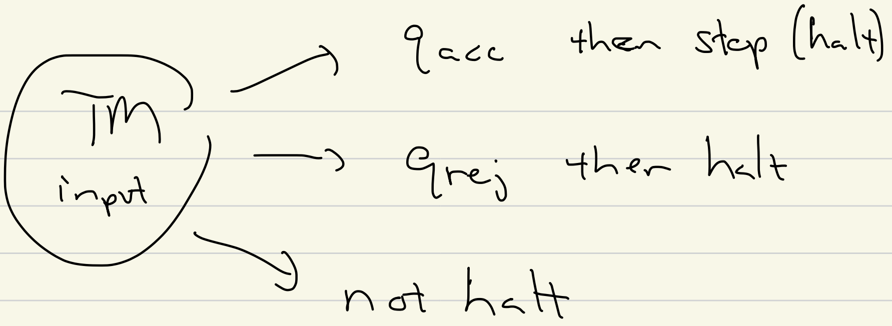


input! 01

[Sip]

Initially! (Time 0)	q_0	0 | 1 | ⊔ | ⊔ | ... cell 1 cell 2	q_0 0 1
Time 1 :	$q_{move\ right}$	X | 1 | ⊔ | ⊔ | ...	X $q_{move\ right}$ 1
Time 2 !	$q_{move\ right}$	x | 1 | ⊔ | ⊔	x 1 $q_{move\ right}$
Time 3	$q_{rightmost}$	x | 1 | ⊔ | ...	X $q_{rightmost}$ 1
Time 4	$q_{move\ left}$	X | X |	$q_{move\ left}$ X X
Time 6	q_{acc}	Done!	Done!





Given a TM, M,

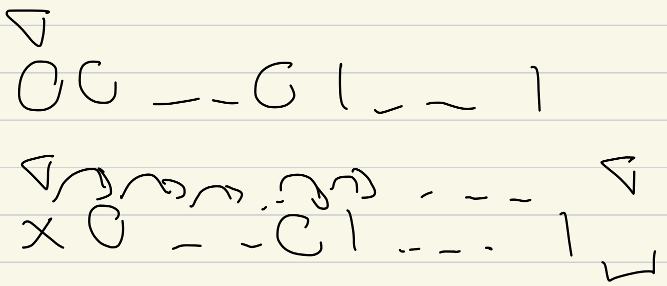
M <u>recognizes</u> $\left\{ s \in \text{input} = \Sigma^* \;\middle|\; \begin{array}{l} \text{on input } s, M \\ \text{reaches } q_{acc} \end{array} \right\}$

We say that M is a <u>decider</u> if on all inputs, M reaches either q_{acc} or q_{rej} (i.e., in a finite time/steps)

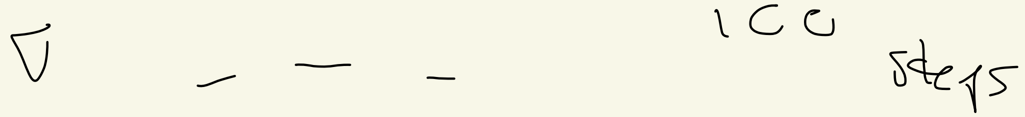
M <u>decides</u> L if $\left\{ \begin{array}{l} M \text{ is a decider and} \\ M \text{ recognizes } L \end{array} \right.$

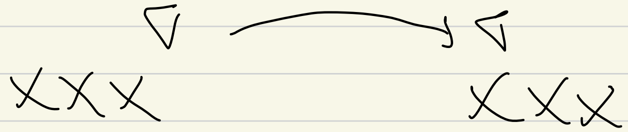
$L = \{0^n 1^n \mid n \in \mathbb{N}\}$, recognize L with a "2-tape TM"

Motivate: The string $0^{50} 1^{50}$, on the TM above



100 steps



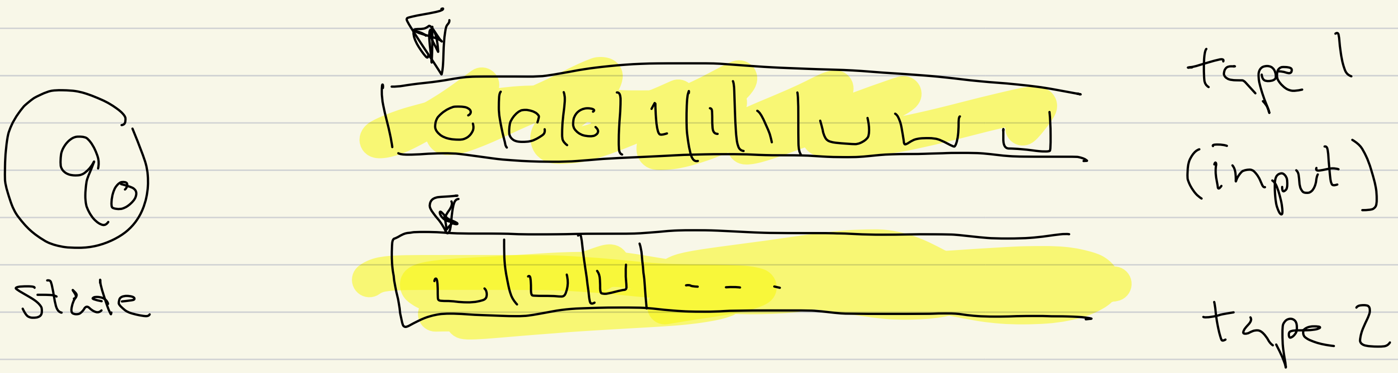


$\#$ steps $\leq 100 \cdot 100$

maybe $\leq 100 + 100 + 98 + 98 + \ldots$

quadratic in size input

2-tape:



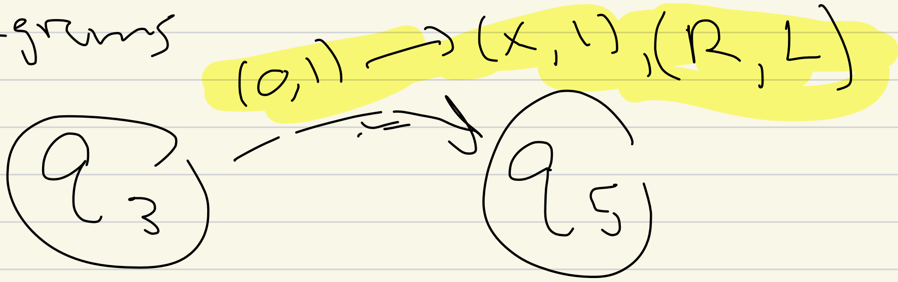
rules: 2-tapes, input appears on tape 1

initially in q_0

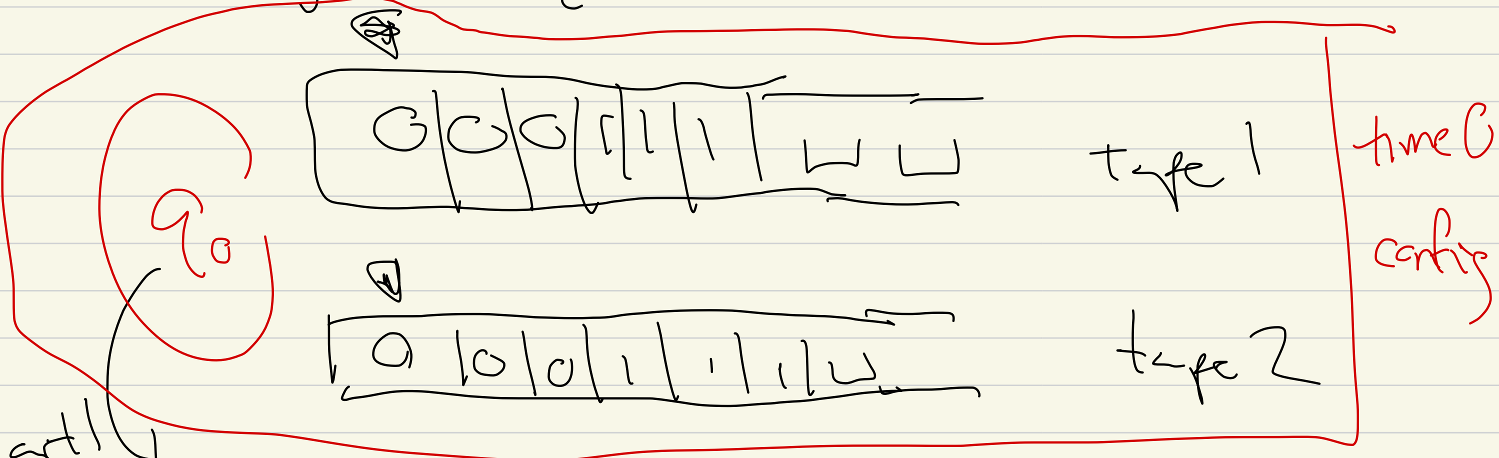
$$\delta : Q \times \Gamma^2 \to Q \times \Gamma^2 \to \{L, R, S\}^2$$

S: tape head stays, doesn't move

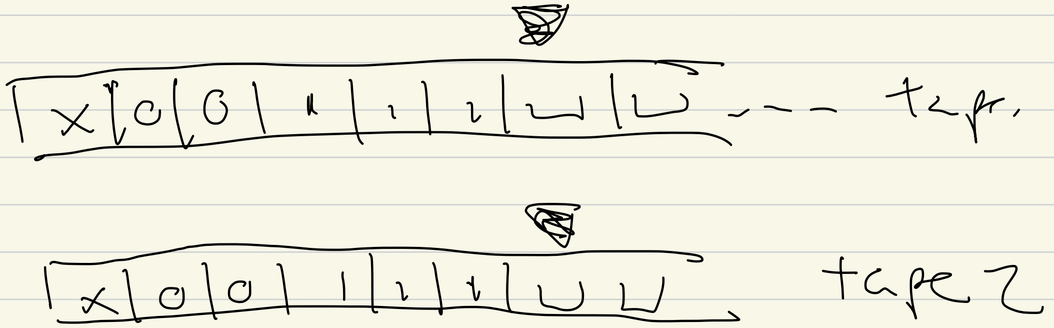
diagrams

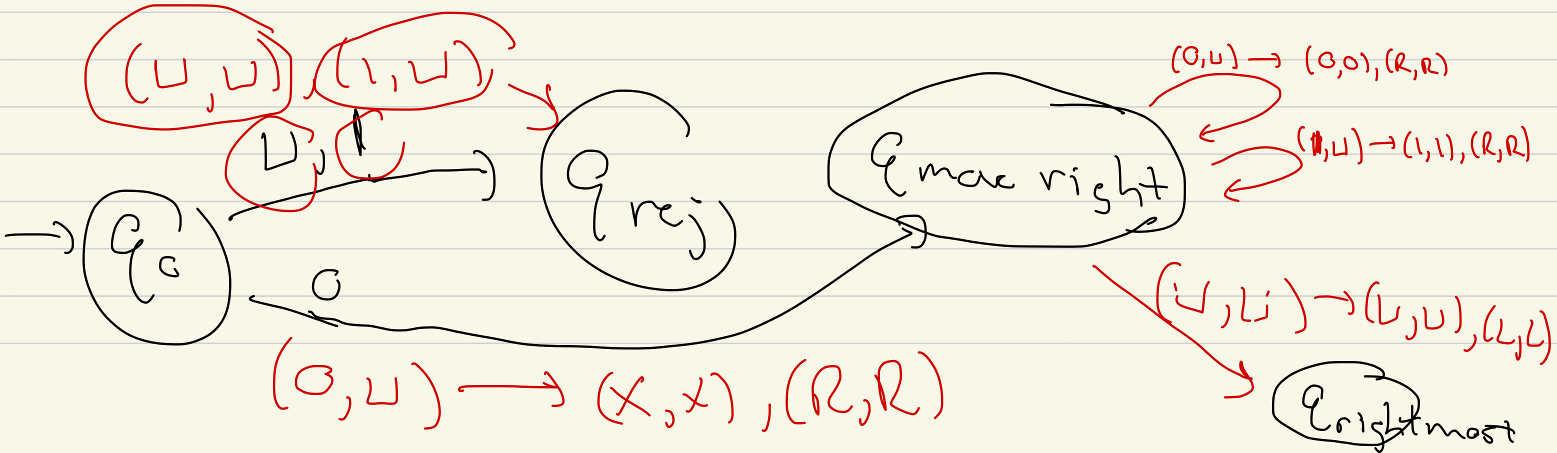


Claim: There is a linear time alg to recognize $\{0^n 1^n \mid n \in \mathbb{N}\}$:

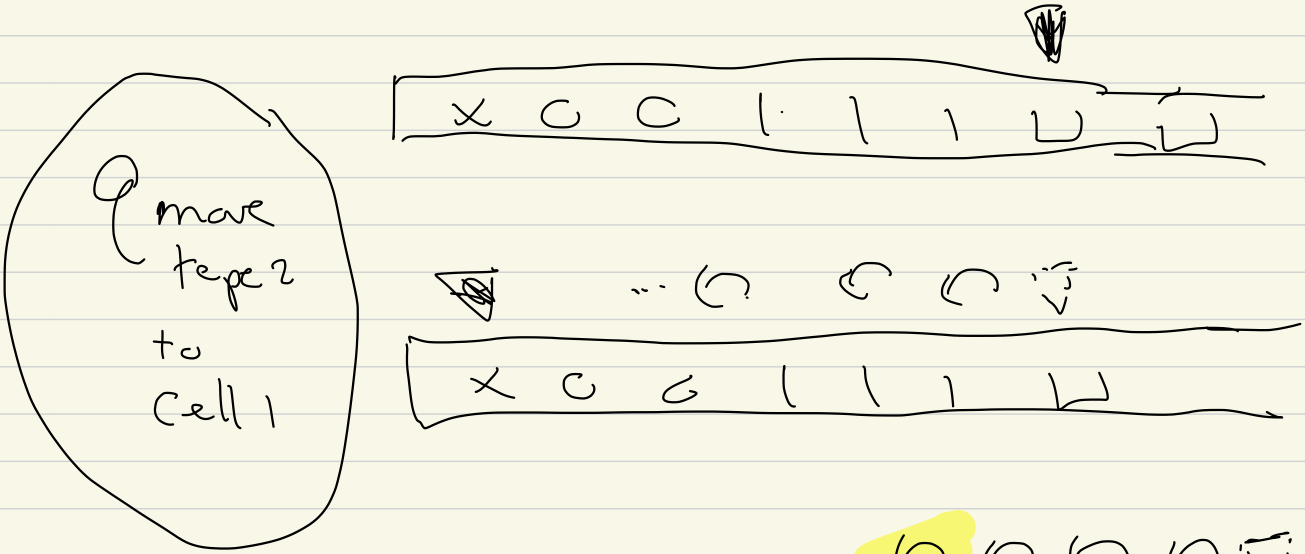


partly of algorithm:

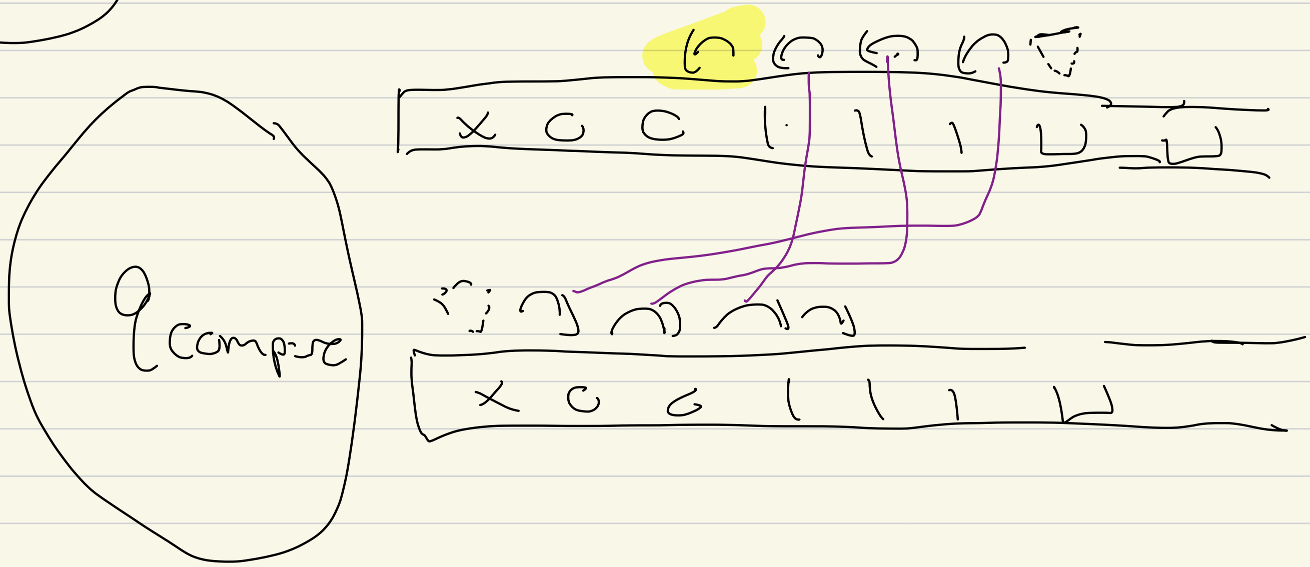




② Send only tape head 2 back to cell 1



③



make sure that 0 on tape 2
matches 1 on tape 1

Claim 2: Any 2-tape TM algorithm
has a "corresponding" 1-tape TM algorithm

Breakout Room Problems ①, ④, ⑤

10:34 → 10:44

① Give high-level or implementation level of Turing machine to decide:

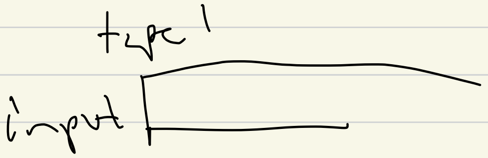
$PRIMES = \{2,3,5,7,11,13,17,\ldots\} \subset \{0,1,\ldots,9\}^*$

$TIMES = \{ a\#b\#c \mid a,b,c \in \{0,1\}^*, \; a \cdot b = c \text{ as base 2 numbers} \}$

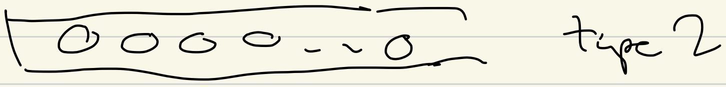
$\Sigma = \{0,1,\#\}$

④, ⑤ $\{TM\text{'s}\}$ countable? $\{TM \text{ algorithms}\}$ countable?

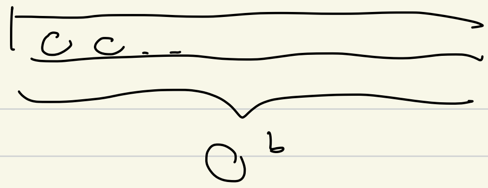
TIMES: Idea ① $a \rightarrow \underbrace{XX\ldots X}_{X^a}$ or $\underbrace{0\ldots0}_{0^a}$



$a \rightarrow 0^a$

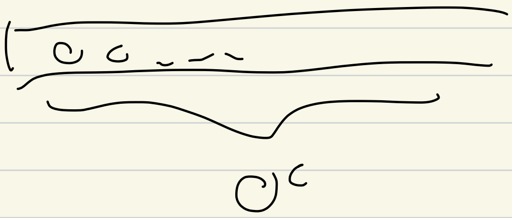


0^b



tape 3

0^c



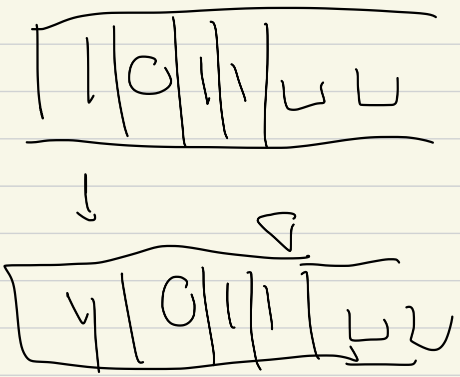
tape 4

tape 5 "binary counter"

for $0^a, 0^b, 0^c$

step 2 & 4 together ^{R}, at end step tape 3 by one ^{R}

tape 5



subtract 1 at a time

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① Build "subroutine" TM to compute a+b

② Then use for a∘b

input 11 # 101 # 11001

↳

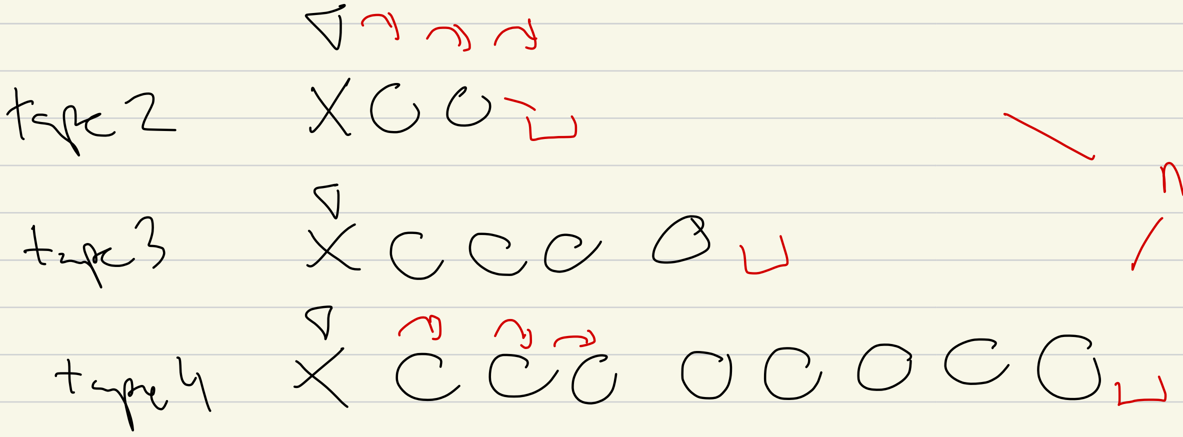
: 3

X	0	0	⊔	⊔	- - -

OR

x'	x	x

write OOO, but "remember" left edge



move R until ⊔ type 2

then move type 2 to left edge

" " 3 by one R

leave type 4 alone (maybe one cell back)