CPSC 421/501, Oct 8, 2020

Update to Zoom 5.3.0 or later to choose your own breakout rooms

- Myhill-Nerode:
  - $\{0^n 1^n\}$ is nonregular
- Implication:

  $\{0^n 1^m 0^p \mid n+p=m\}$ is non-regular

  (intersect with regular language $0^* 1^*$)

- Myhill-Nerode
  - Min DFA for $L = a^{23} a^*$
  - " " " $L = a^{23} a^* \cup \{a^0, a^1, a^5\}$

Don't forget to carefully argue that sets are distinct: e.g.

$\{\text{odd perfect numbers}\}$ vs. $\emptyset$

$\{0,1\}^*$ vs. $(\varepsilon \cup 1)(1,0)^*(\varepsilon \cup 0)$

Breakout Room Problems:

① Show that $L = \{ 0^{n^2} \mid n \in \mathbb{Z} \}$ is nonregular

$= \{ 0^1, 0^4, 0^9, 0^{16}, \ldots \}$

② Show that $L = \left\{ 0^n 1^m \;\middle|\; \begin{array}{l} n, m \in \mathbb{Z} \\ n \geq 2m \end{array} \right\}$

is non-regular

③ Show that

$L = \{ w \in \{a,b\}^* \mid$ w has the same number of a's as b's $\}$

is non-regular

④ If $w = \sigma_1 \ldots \sigma_k$, then $w^{\text{reverse}} = \sigma_k \ldots \sigma_1$

e.g. $(abb)^{rev} = bba$

Show that

$$\text{PALINDROME} = \{ w \in \{a,b\}^* \mid w = w^{rev} \}$$

is non-regular.

5 Give a DFA with the few possible states accepting $(aab, ab)^*$:

5a Give the DFA

5b Use Myhill-Nerode to prove that your DFA has the fewest possible states.

Today finish Myhill-Nerode and §1.4 [Sip]
on nonregular languages [we are skipping the
"pumping lemma"].

Claim: The language $\{0^n 1^n \mid n \in \mathbb{N}\}$,
i.e. $\{01, 0011, 0^3 1^3, 0^4 1^4, \ldots\}$ is nonregular, i.e.
is not recognized by any DFA.

Intuition: When you see $00\ldots0$ as input
you have to "count" how many 0's you seen

Proof: $\mathrm{AccFut}_L(u) = \{v \in \Sigma^* \mid uv \in L\}$

① $\mathrm{AccFut}_L(\varepsilon) = L = \{01, 0^2 1^2, 0^3 1^3, \ldots\}$

$\mathrm{AccFut}_L(0) = \{v \mid 0v \in L\}$
$= \{1, 011, 0^2 1^3, \ldots\}$

$\mathrm{AccFut}_L(00) = \{11, 01^3, 0^2 1^4, \ldots\}$

⋮

$AccFut_L(0^k) = \{1^k, 01^{k+1}, 0^2 1^{k+2}, \ldots\}$

⋮

(2) For any k, all set $AccFut_L(0^k)$ are distinct. Why:

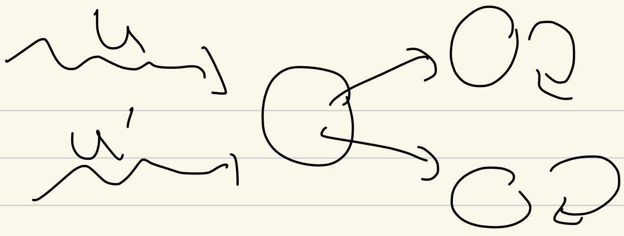
- $AccFut_L(0^k)$ contains 1^k but no other element of 1^*

So if $k \neq k'$ $\begin{cases} AccFut_L(0^k) \text{ contains } 1^k \text{ not } 1^{k'} \\ AccFut_L(0^{k'}) \quad " \quad 1^{k'} \text{ not } 1^k \end{cases}$

- The short string in $AccFut_L(0^k)$ is of length k. (i.e. 1^k)

(3) If there are infinitely many sets of the form $AccFut_L(u)$ as u varies over $u \in \Sigma^*$, then L is non-regular

Recall: if u, u' fall in same DFA state



then

$\mathrm{AccFut}_L(u)$

$= \mathrm{AccFut}_L(u')$

Myhill-Nerode thm.

One more tool! L

① Say we know $\{0^n 1^n\}$ is nonregular.

Claim: $L' = \{w = 0^n 1^n 0^m$ s.t.

② $n \in \mathbb{N},\ m = 0, 1, 2, \dots\}$

is non-regular

$\{0^n 1^n\} \subset \{0,1\}^*$

non-regular reg

$w \in \{0^n 1^n\} \Rightarrow w \in \{0^n 1^n 0^m\}$

L L'

(a) Still show $Accfut_L(0^k)$ are all distinct

(still shortest element of $Accfut_L(0^k)$ is length k)

(b) Say that $L' = \{0^n 1^n 0^m \mid \begin{smallmatrix} n \in \mathbb{N} \\ m = 0,1,\ldots \end{smallmatrix}\}$ is regular. Then

$$\underset{\text{reg}}{L'} \cap \underset{\text{reg}}{0^* 1^*} = \underbrace{\{0^n 1^n\}}_{\text{we know this is not regular}}$$

Argument (b) takes non-regular languages (proven somehow) and gives you more non-regular languages.

Remark: If L' is non-regular

$L' \subset \Sigma^*$, and Σ^* is regular,

but if you want to prove that

L' is non-regular, and

$$L' \cap \begin{pmatrix} \text{some known} \\ \text{regular} \\ \text{language} \end{pmatrix} = \begin{pmatrix} \text{some known} \\ \text{nonregular} \\ \text{language} \end{pmatrix}$$

$\Rightarrow L'$ is nonregular

If L' is nonregular $\Rightarrow \Sigma^* \setminus L'$ is nonregular

since reverse final states with nonfinal states

$$\left(\{0^n 1^n\}\right) \cup \left(\{0,1\}^* \setminus \{0^n 1^n\}\right) = \{0,1\}^*$$

non-regular   non-regular   regular

$$\left(\{\varepsilon\} \cup \{0^n 1^n\}\right) \circ \left(\{\varepsilon\} \cup \underbrace{\left(\{0,1\}^* \setminus \{0^n 1^n\}\right)}_{\text{non-regular}}\right)$$

nonregular

$$= \{0,1\}^*$$

$$\left(\{\varepsilon\} \cup L\right) \circ \left(\{\varepsilon\} \cup (\Sigma^* \setminus L)\right) = \Sigma^*$$

$$\varepsilon \circ (\Sigma^* \setminus L)$$

OR

$$L \circ \varepsilon$$

{ odd perfect numbers }

perfect numbers :

$\{ n \mid$ sum of divisors of n less than n equals $n \}$

6 divisible by 1, 2, 3, (6)

$1+2+3=6$

28 divisible by 1, 2, 4, 7, 14

$1+2+4+7+14=28$

$\{$odd perfect numbers$\} \overset{\text{conj}}{=} \emptyset$

not clear if equal

$L = a\{a,b\}^*$

$\mathrm{AccFut}_L(b) = \emptyset$

but

$(\{a,b\}^* \setminus \{a^n b^n\}) \cap \{a^n b^n\}$

$= \emptyset \cap \{\text{anything}\} = \{a^n \{a,b\}^* \mid n \text{ is an even prime number} > 2\}$

reg exp$_1$ reg exp$_2$

these could describe same language even if these expressions are very different

Suggest Probs 1 & 5 of breakout problems

10:22 $\longrightarrow$ 10:32

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1 Show $\{0^{n^2} \mid n \in \mathbb{N}\} = L$

$= \{0, 0^4, 0^9, 0^{16}, \ldots\}$

is nonregular

Next Thursday I'll reserve Rooms 1–? for random groups

$L' = \{0^{n_1}, 0^{n_2}, 0^{n_3}, \ldots\}$ $n_1, n_2, \ldots$ increasing

(

more general

then

$$\mathrm{Accfut}_{L'}(0^{n_i}) = \{\varepsilon, 0^{n_{i+1}-n_i}, \ldots\}$$

$$\mathrm{Accfut}_L(0) = \{\varepsilon, 0^{4-1} = 0^3, 0^{9-1}, \ldots$$

$$\mathrm{Accfut}_L(0^4) = \{\varepsilon, 0^{9-4} = 0^5, \ldots$$

$$\mathrm{Accfut}_L(0^9) = \{\varepsilon, 0^{16-9} = 0^7, \ldots$$

Claim: 2^{nd} smallest string in

$\mathrm{Accfut}_L(0^{k^2})$ is 0^{2k+1}

So if $k \neq k'$, $0^{2k+1} \neq 0^{2k'+1}$

hence $\mathrm{Accfut}_L(0^{k^2}) \neq \mathrm{Accfut}(0^{(k')^2})$

By contrast $\{\varepsilon = 0^0, 0^7, 0^{14}, 0^{21}, 0^{28}, \ldots$

is regular

L'

$\text{AccFut}_{L}(a^k)$ depends only on $k \bmod 7$

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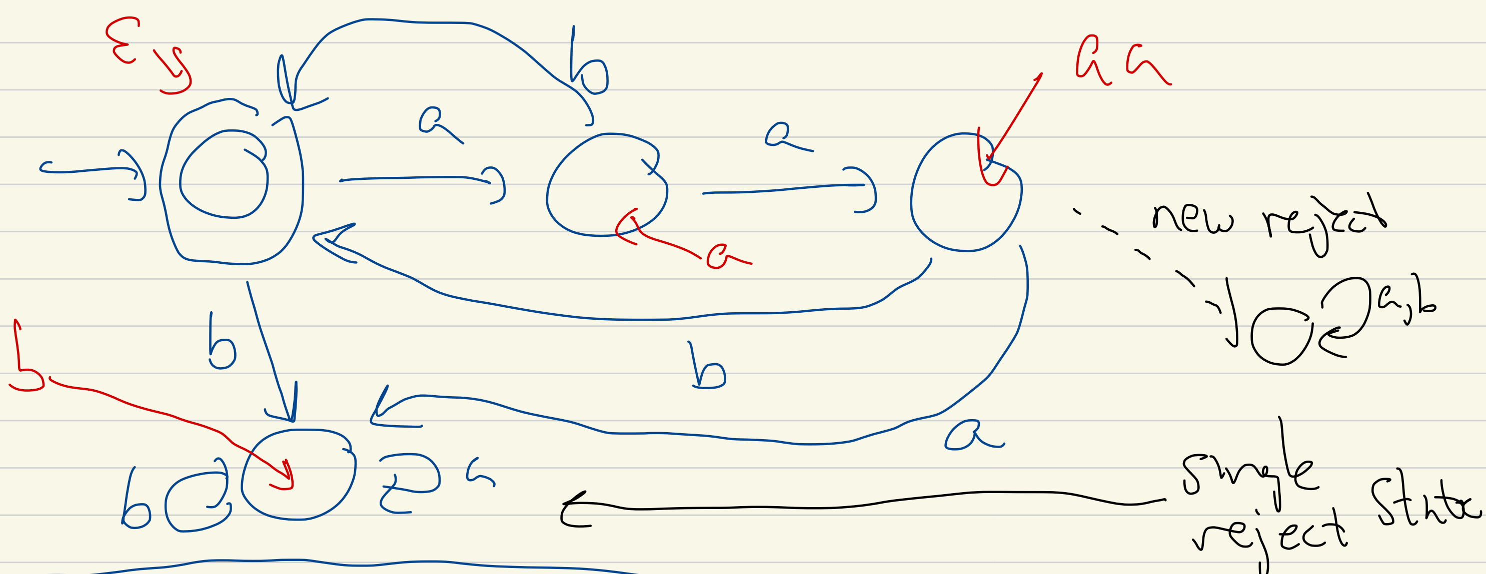
(5) $(aab, ab)^*$: build a DFA for language & use M-N thm to show that you DFA has fewest number of states



$(aab, ab)^* = \{\varepsilon, ab, aab, abab, \dots\}$

$$\mathrm{AccFut}_L(\varepsilon) = \{\varepsilon, ab, aab, \ldots\}$$

$$\mathrm{AccFut}_L(a) = \{b, ab, \ldots\}$$

$$\mathrm{AccFut}_L(aa) = \{b, bab, baab, \ldots\}$$

$$\mathrm{AccFut}_L(b) = \emptyset$$

← distinct, since all others non-empty

distinct, since contains ε

shortest string is "b"

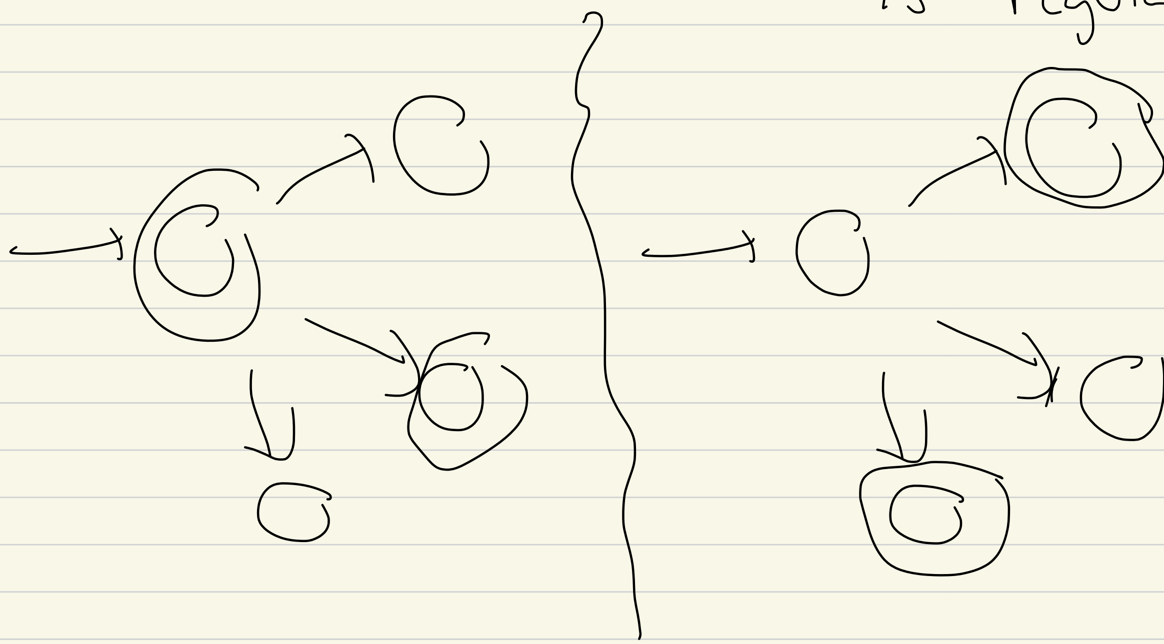
$$\mathrm{AccFut}_L(a) \neq \mathrm{AccFut}_L(aa)$$

ab in here

not in here

End of class

If L is regular $\iff$ $L^{comp} = \Sigma^* \setminus L$ is regular

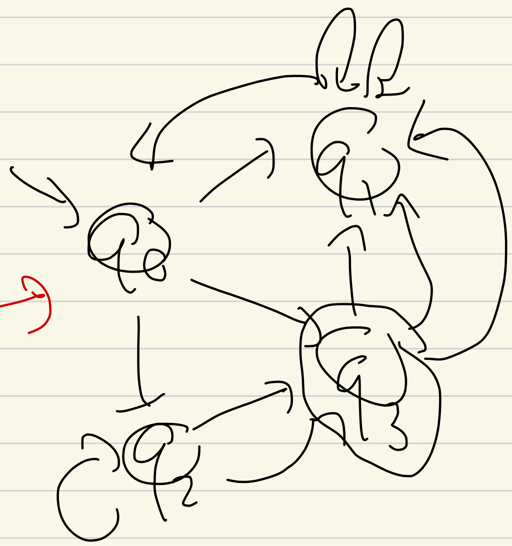


Thm: If L is non regular $\Rightarrow$ $\Sigma^* \setminus L$ is non regular

Pf: If L is non-regular, but $\Sigma^* \setminus L$ is regular $\Rightarrow$ $\underbrace{\Sigma^* \setminus (\Sigma^* \setminus L)}_{L}$ is regular

$$L^{comp} = \Sigma^* \setminus L$$

$$(L^{comp})^{comp} = L$$

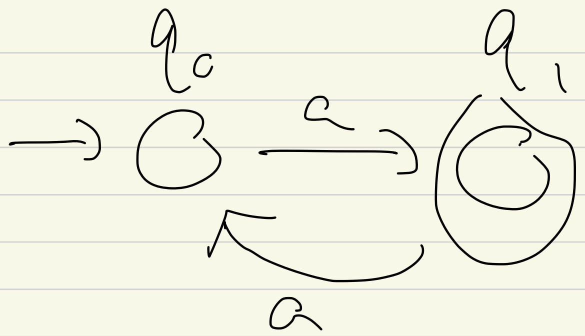


$\{ w \in \Sigma^* \mid$ follow w thru DFA get to $q_0 \}$

if wu leads to final/accepting state

and w, w' both lead to q_0

$\Rightarrow$ $w'u$ leads to final/accepting state



accept at q_1

$$L = \{ a, a^3, a^5, \ldots \}$$

If a^6 and $a^4 \longrightarrow q_0$

then for any $w \in \{a\}^*$

either both $a^6 w$ and $a^4 w$ final state

or " " " " non-final state

$$\mathrm{AccFut}_L(a^4) = \{ w \mid a^4 w \in L \}$$

$$= \mathrm{AccFut}_L(a^6) = \{ w \mid a^6 w \in L \}$$