CPSC 421/501

- Continue § 1.1 of [Sip]

  - The "cost" of excluding ε from $\{0,3,6,9,00,03,06,09,12,15,\ldots\}$

  - More on DFA's

  - $\underbrace{A \cup B,\ A \cap B,\ A \circ B,\ A^*}$ } take regular languages, give new regular languages

  - Why $A \circ B$ and A^* are awkward with DFA's

    e.g. $\{a^3, a^5\}^*$

- Start § 1.2 of [Sip]

  - NFA's : non-determinism

  - How they help with $A \circ B$, A^*

Breakout Room Questions!

(1) Give a DFA that recognizes

(a) $\{\varepsilon, 0, 2, 4, 6, 8, 00, 02, 04, 06, 08, 10, 12, 14, \ldots\}$

(b) Same, but exclude ε

(c) " " " ε and do not allow leading 0's

(2) Give a DFA that recognizes $\{0, 3, 6, 9, 12, 15, 18, 21, \ldots\}$

(3) Is there a DFA that recognizes $\{0, 7, 14, 21, 28, 35, 42, \ldots\}$

(4) How many states needed to recognize $\{a^5, a^7\}$ by a DFA

(5) How many states needed to recognize $\{a^5, a^7\}^*$ by a DFA

(6) How many states needed to recognize $\{a^5, a^7\}^*$ by an NFA

(7) If an NFA has 1000 states, its corresponding DFA may have roughly 2^{1000} states. Is there a relatively quick way to see if the NFA accepts a given string?

Canvas Survey: Roughly 40 students responded

- About $\frac{1}{2}$ wanted no breakout rooms (mostly 5 min break)
- " $\frac{1}{4}$ " randomized breakout rooms
- " " " student selected " "

[Preference you can express?

Mostly covering textbook [Sip], sometimes handouts

For DFA's, look at [Sip], Exercises 1.1–1.6

for DFA's, § 1.1

e.g. 1.5 A a
A b
c
d
⋮

A = Answers

Review DFA's = finite automata, D = deterministic

L over $\Sigma = \{a, b\}$

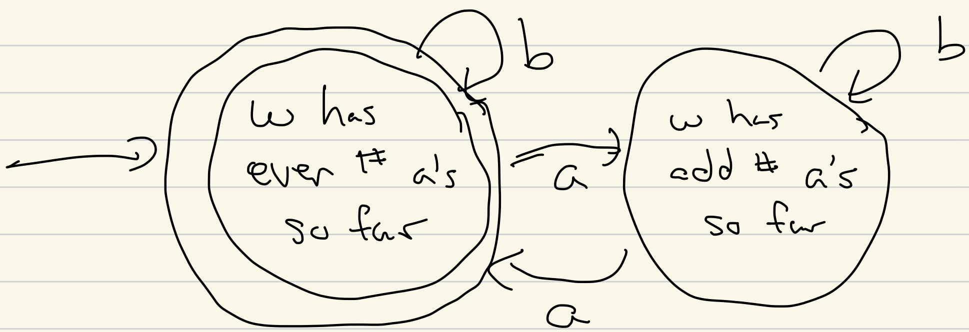
Take $L = \{ a^n b^n \mid n \in \mathbb{N} \} = \{ ab, aabb, aaabbb, a^4b^4, \ldots \}$

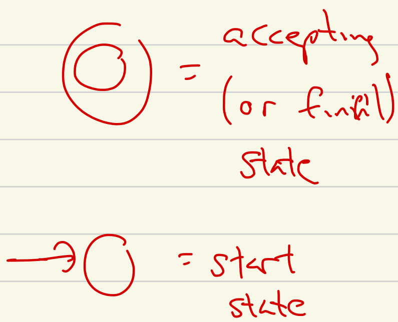
turns out to be non-regular (prove this later)

A language is <u>regular</u> if it is recognized by some DFA.

Example:

$$L_{\text{even } a\text{'s}} = \{ w \in \{a,b\}^* \mid w \text{ has an even \# of } a\text{'s} \}$$





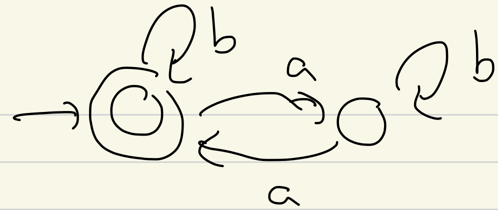
$w = \sigma_1 \ldots \sigma_k, \quad \sigma_i \in \{a,b\}$

start before σ_1

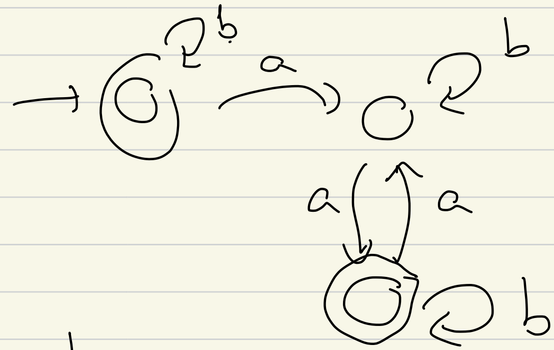
$$L_{\text{even } a\text{'s}} = \{ \varepsilon, aa, aab, aba, baa, aaaa, aabb, \ldots \}$$

$$L_{\text{positive even \# } a\text{'s}} = \{ aa, aab, aba, baa, aaaa, aabb, \ldots \}$$

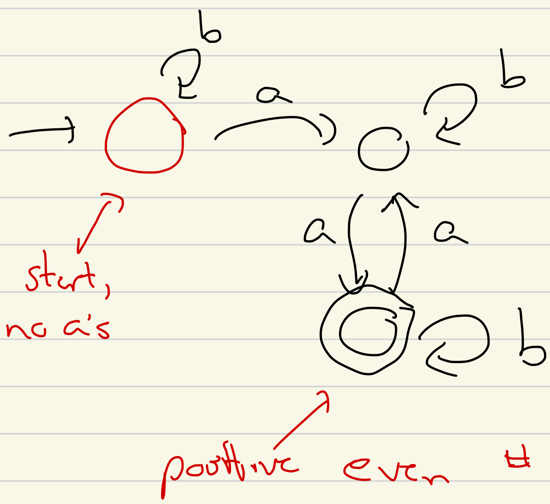
$L_{\text{even } a\text{'s}}$ recognized by



" "



$L_{\text{even positive } \# a\text{'s}}$

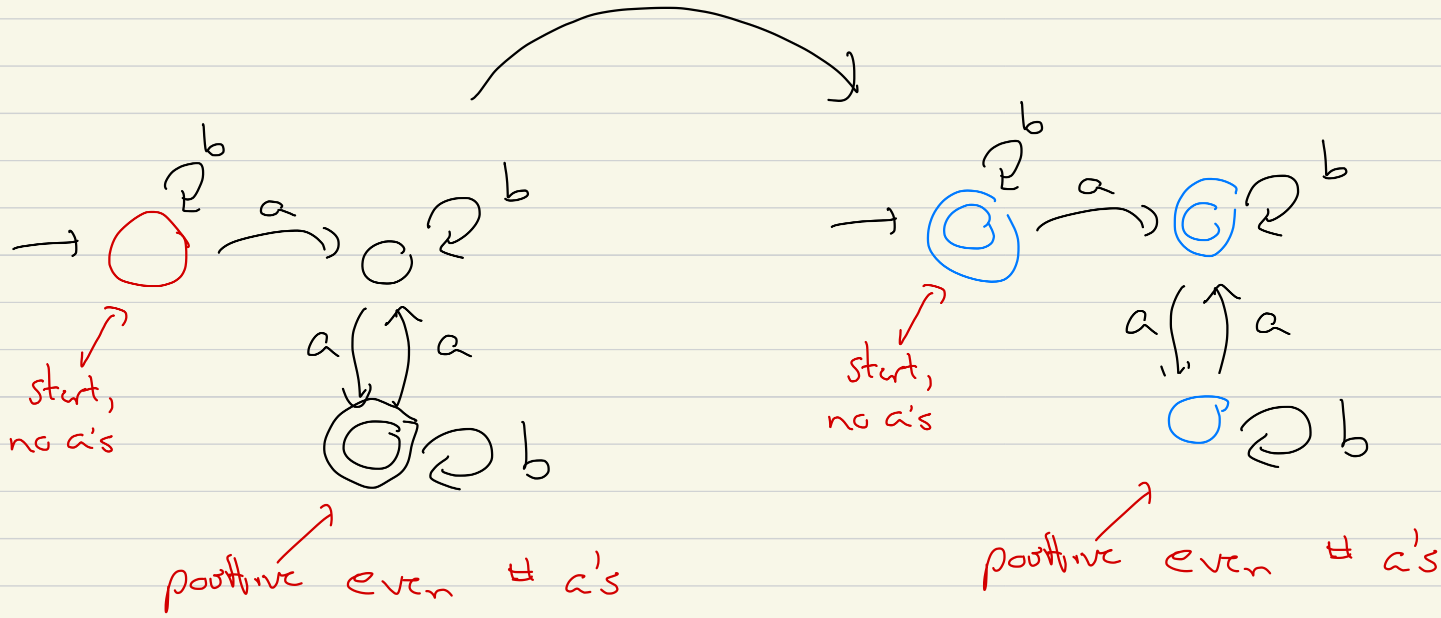


Complement of $L_{\text{even, pos } \# a\text{'s}}$

$\left(L_{\text{even pos } \# a\text{'s}}\right)^{\text{Comp}}$ or $\{a,b\}^* \setminus L_{\text{even pos } \# a}$

Using notation: A, B sets, $A \setminus B = \{ w \in A \mid w \notin B \}$

$$L \longrightarrow L^{\text{comp}}$$



Tells us: L is regular $\Longrightarrow$ L^{comp} is regular

Pf: Switch accepting $\longleftrightarrow$ rejecting states.

Theorems: If A, B languages over Σ

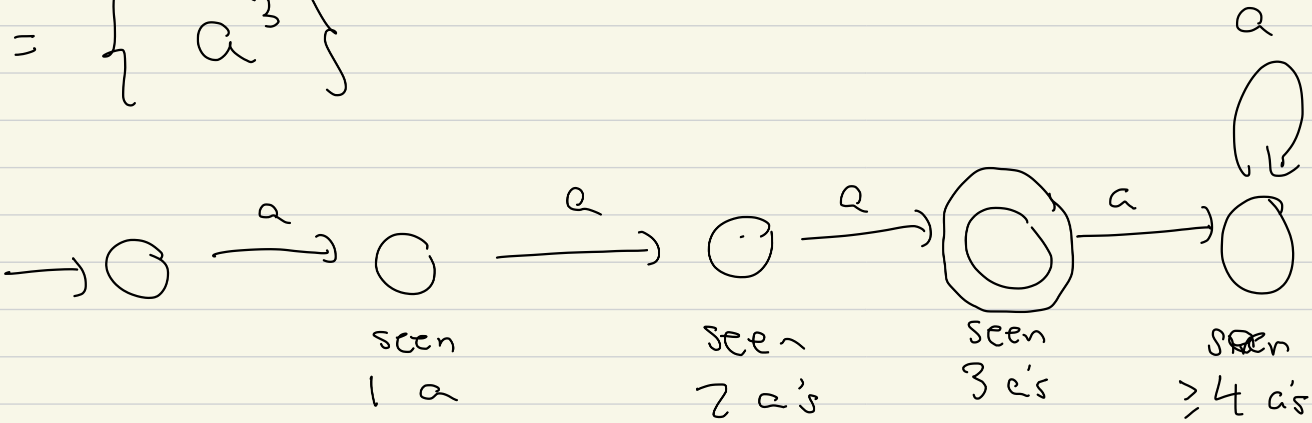
$$A \circ B = \{ w_1 w_2 \mid w_1 \in A,\ w_2 \in B \}$$

$$A^* = \{ w_1 \cdots w_k \mid k = 0, 1, 2, \ldots \text{ and } w_1, \ldots, w_k \in A \}$$

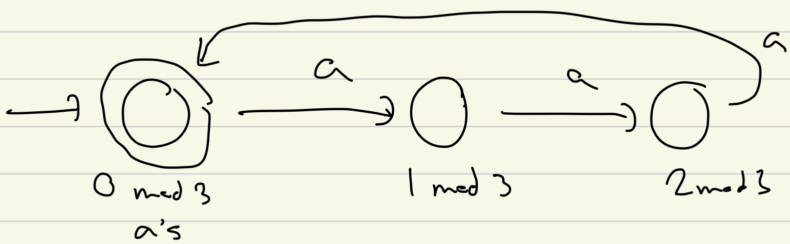
If A, B regular, so is $A \circ B$, A^*.

Example: $\Sigma = \{a\}$.

$L = \{a^3\}$



$L^* = \{\varepsilon, a^3, a^3 \circ a^3 = a^6, a^9, a^{12}, \ldots\}$



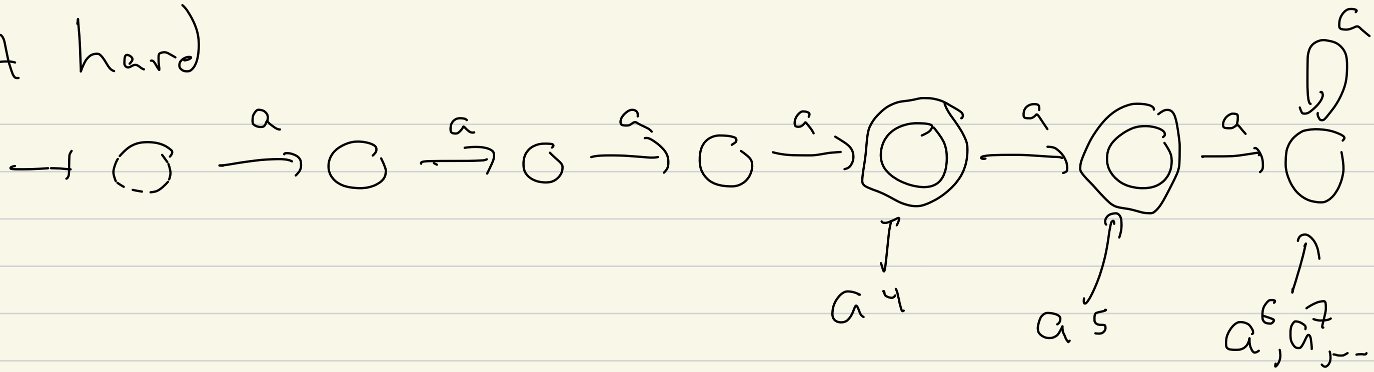
recognizes $\{a^3\}^*$

$\Sigma = \{a\}$, $L = \{a^4, a^5\}^*$

$L = \{\varepsilon, a^4, a^5, a^4a^4, a^4a^5, a^5a^4, a^5a^5, \ldots\}$

To recognize $\{a^4, a^5\}$ with DFA is

not hard



$$L = \{ \varepsilon, a^4, a^5, a^8, a^9, a^{10}, \underbrace{a^{12}}_{=(a^4)^3}, a^{13}, a^{14}, a^{15}, a^{16}, \ldots \}$$

not in L: $a, a^2, a^3, a^6, a^7, a^{11}$

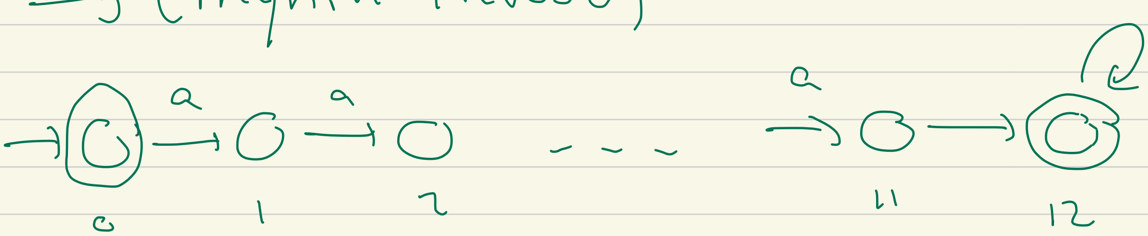
You can show:

$$L = \{ \varepsilon, a^4, a^5, a^8, a^9, a^{10}, \underbrace{a^{12}, a^{13}, \ldots}_{\text{everything } a^n,\ n \geq 12} \}$$

Write a DFA to recognize this!

Claim: $a^{11} \notin L$ but $a^n \in L$ for $n \geq 12$

$\Rightarrow$ (Myhill-Nerode)



L^* tricky, # states to recognize L^* can be much larger than to recognize L

Also

$$\left(\underbrace{\{a^3\}^*}_{\{\varepsilon, a^3, a^6, a^9, \dots\}}\right) \circ \left(\underline{\{a^5\}^*}\right) = \{a^3, a^5\}^*$$

fairly simple

fairly simple

concatenate $\{a^3\}^*$ with $\{a^5\}^*$

$$A \circ B = \{ w_1 w_2 \mid w_1 \in A,\ w_2 \in B \}$$

If $w \in A \circ B$ then for some k,

$w = \sigma_1 \dots \sigma_k$, then $\sigma_1 \dots \sigma_\ell \in A$

$\sigma_{\ell+1} \dots \sigma_k \in B$

but you are not told what is ℓ.

For this reason: introduce

NFA (§1.2 [Sip])

↑

non-deterministic finite automata

We will use NFA to show

A, B regular, then $A \circ B$, A^*

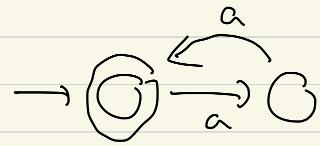
are also regular.

Break 10:26:45

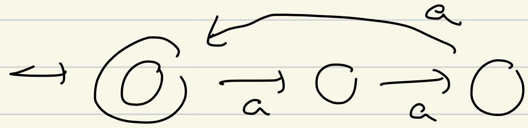
10:31:45

Idea: $L = \{a^2\}^* \circ \{a^3\}^*$

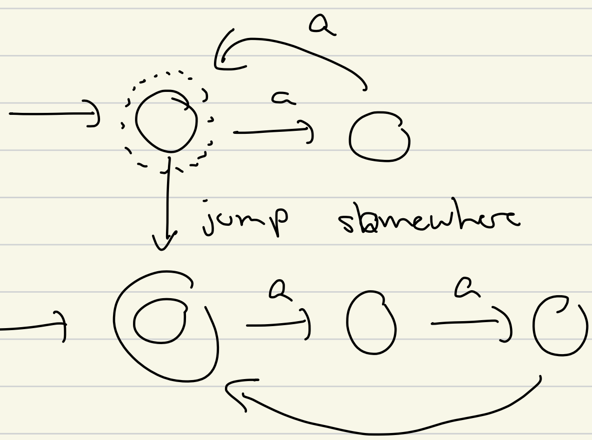
Accept $\{a^2\}^* = \{\varepsilon, a^2, a^4, \dots\}$



Accept $\{a^3\}^*$



Start:

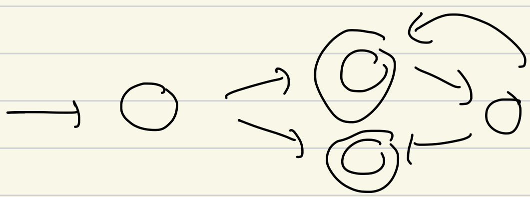


first language

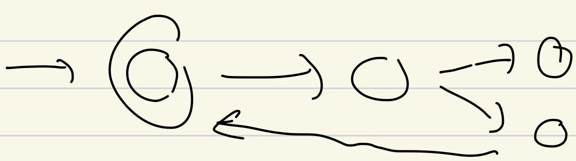
second language

More generally

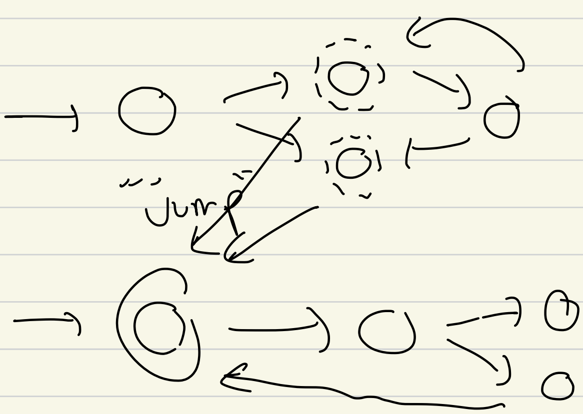
L_1



L_2



$\Rightarrow$



$L_1 \circ L_2$

need = "jump state"

Formally NFA = $(Q, \Sigma, \delta, q_0, F)$

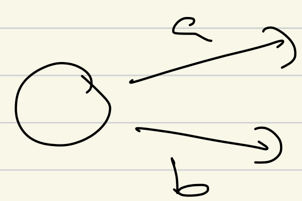
but $\delta : Q \times \Sigma_\varepsilon \rightarrow$ Set of Subsets of (Q)

$\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

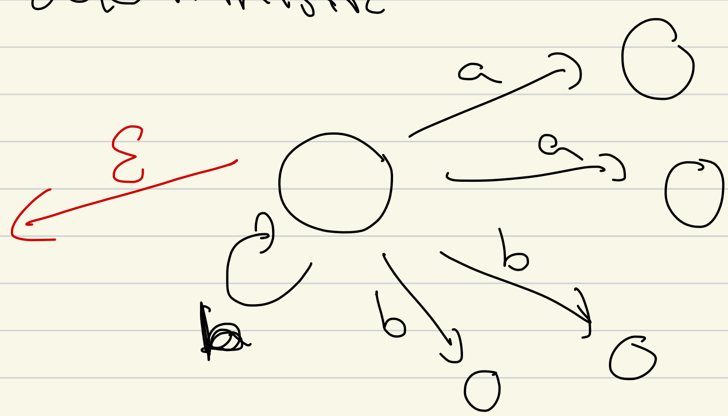
$\delta : Q \times \Sigma_\varepsilon \rightarrow \text{Power}(Q)$

N = non-deterministic

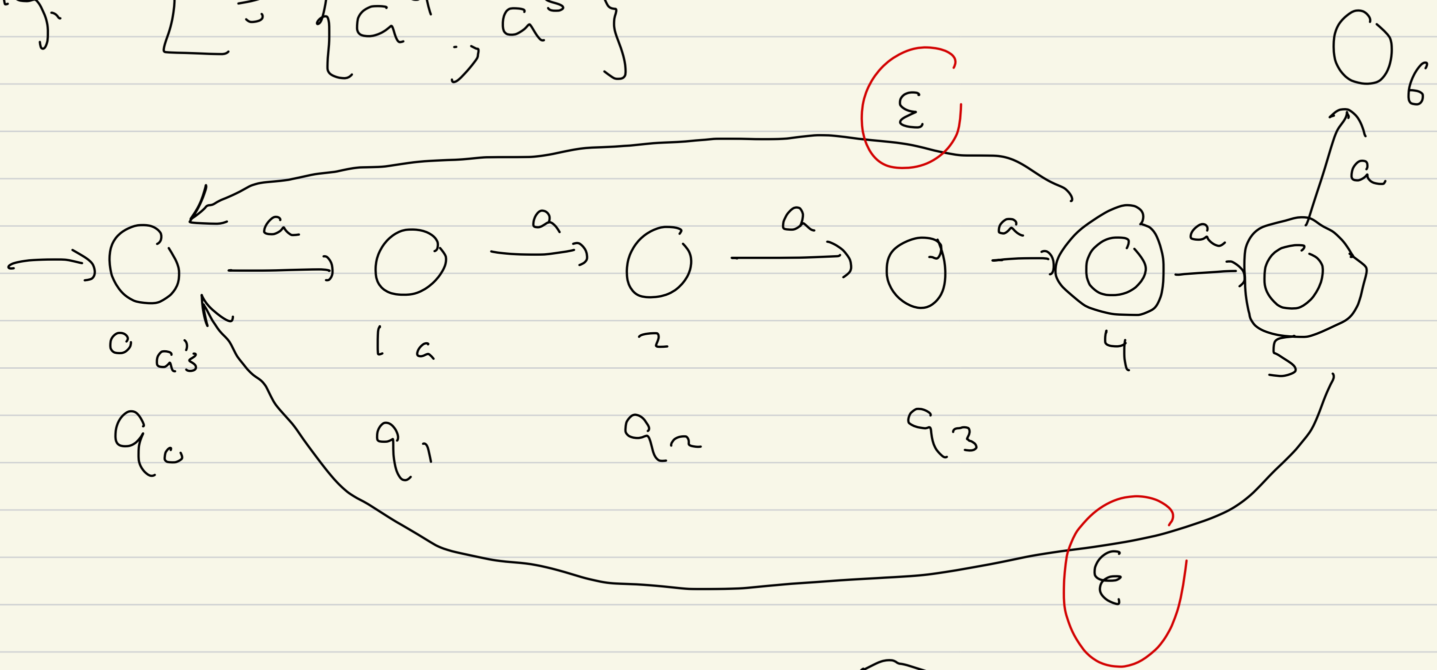
Deterministic



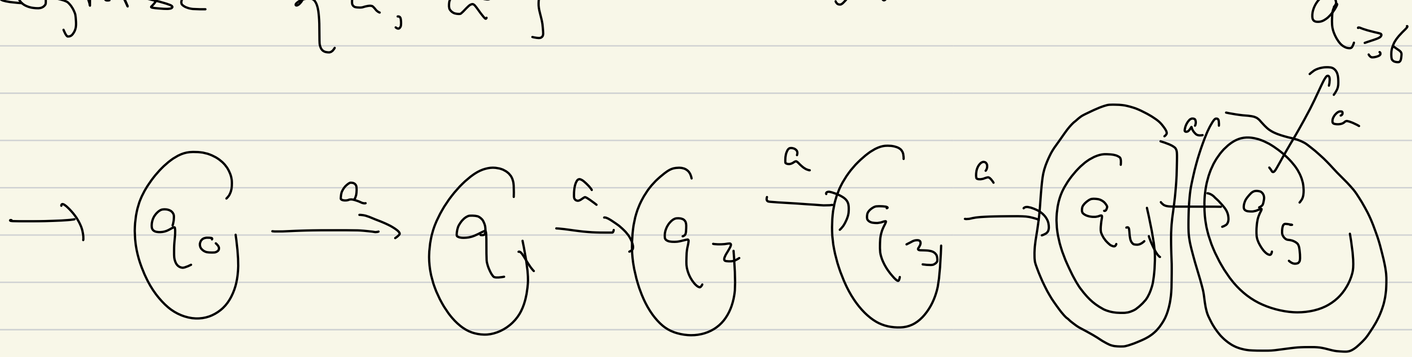
Non-deterministic



where $\xrightarrow{\varepsilon}$ you don't process the next letter e.g. $L = \{a^4, a^5\}^*$



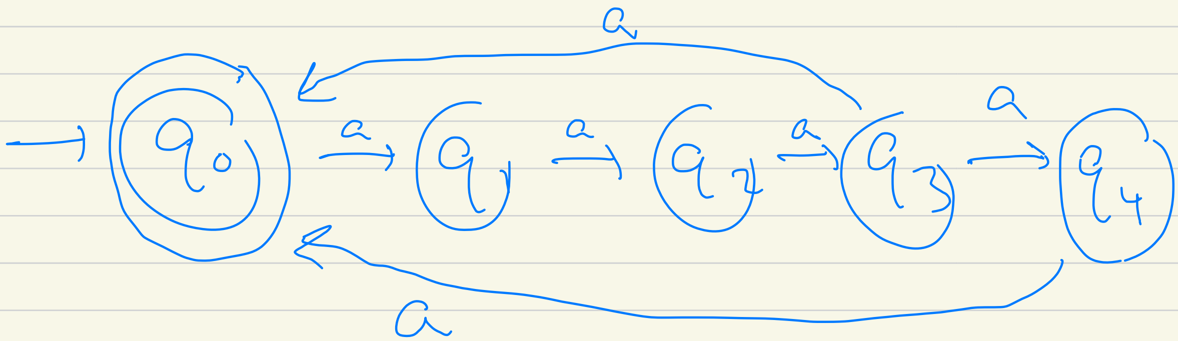
Recognize $\{a^4, a^5\}$

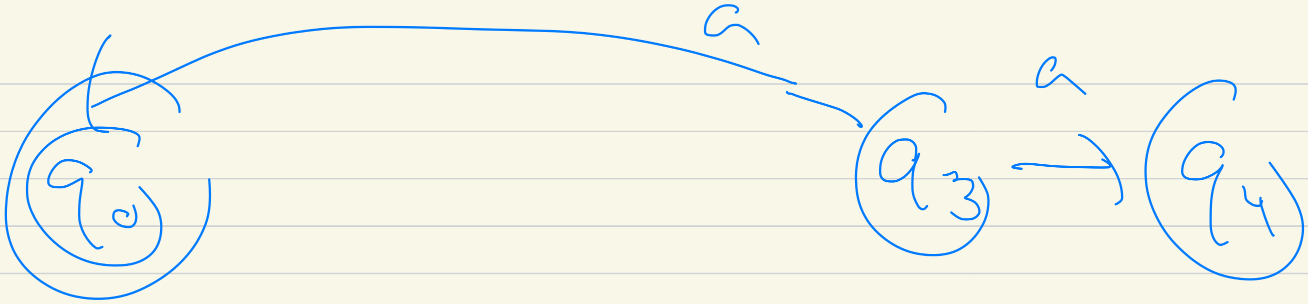


Typically: use jumps when recognize $A \circ B$

A^*

Another NFA for $\{a^4, a^5\}^*$





See at q_3 on a,

can move either from $q_3 \to q_4$

$q_3 \to q_0$

Language recognized by an NFA

is $\{w \mid$ there is some allowable path thru NFA that reaches a final/accepting state$\}$

Next time:

- more examples of NFA
- any languages recognized by an NFA is regular, i.e. " " some DFA.