CPSC 421/501 Sept 24, 2020

Topics: - Set theory remarks

- Resolution of Russell's Paradox
- Some other paradoxes/theorems

Involve "self-reference" + negation

- Start Ch. 1 on Regular Languages [Sip]

Section 1: Finite Automata (DFA's)

- DFA: the idea
- DFA: definition $(Q, \Sigma, \delta, q_0, F)$
- Regular Languages = recognized by some DFA
- $A \cup B$, $A \cap B$, $A \circ B$, A^*
- Why NFA's? [Answer: for $A \circ B$, A^*]

Section 2: Non-deterministic Finite Automata (NFA's)

CANVAS SURVEY AT END

Breakout Room Questions:

1. Come up with your own paradox (related to those of §5,6 of Handout)

2. Give a DFA that recognizes

$$\{0,3,6,9,12,15,\ldots\} \subset \{0,\ldots,9\}^*$$

3. Give a DFA that recognizes

$$\{0,3,6,9,03,06,09,12,15,\ldots\}$$

4. Give a DFA that recognizes

$$\{\varepsilon,0,3,6,9,03,06,09,12,15,\ldots\}$$

5. Is there a DFA that recognizes

$$\{0,7,14,21,28,35,42,\ldots\}$$

6) Give a DFA that recognizes

$\{1^5, 1^7\} \subset \{1\}^*$

7) Give a DFA that recognizes

$\{1^5, 1^7\}^* \subset \{1\}^*$

Last Time:

(4) Is the set of functions $\mathbb{N} \to \mathbb{N}$ that you can describe ~~by~~ a finite string in English: countable or uncountable?

$$\text{ASCII}^* \longrightarrow \{\text{meaningless as function}, \text{ or some function}\}$$

(5) Yet, $\{\mathbb{N} \xrightarrow{\text{functions}} \mathbb{N}\}$ is uncountable; proof!

let's give a surjection $\{\mathbb{N} \xrightarrow{\text{functions}} \mathbb{N}\} \longrightarrow \{\mathbb{N} \xrightarrow{\text{functions}} \{\text{yes}, \text{no}\}\}$

$$f : \mathbb{N} \to \mathbb{N} \longmapsto \tilde{f}(n) = \begin{cases} \text{yes, if } f(n) = 1, \\ \text{no, if } f(n) \neq 1 \end{cases}$$

$$\left(\text{functions } \mathbb{N} \to \{\text{yes}, \text{no}\}\right) \xleftrightarrow{\text{bij}} \text{Power}(\mathbb{N}) \leftarrow \text{uncountable}$$

$\boxed{\text{Fact: If have surjection } S \to T \text{ and } T \text{ is uncountable, then } S \text{ is uncountable}}$

(1) Breakout $\rightarrow$ HW

$$\{\text{function } A \to \mathbb{N}\} \xrightarrow{\text{surj}} \{\text{functions } A \to \{\text{yes}, \text{no}\}\}$$

$f \longmapsto \hat{f}$ given by $\tilde{f}(a) = \begin{cases} \text{yes} & \text{if } f(a)=1, \\ \text{no} & \text{if } f(a) \geq 2 \end{cases}$

really $\mathbb{N} \xrightarrow{\text{surj}} \{\text{yes}, \text{no}\}$

funct $A \xrightarrow{f} \mathbb{N} \quad \rightsquigarrow \quad A \xrightarrow{f} \mathbb{N} \xrightarrow{\text{surj}} \{\text{yes}, \text{no}\}$

Cantor's Thm: Takes $S \xrightarrow{f} \text{Power}(S)$

forms $T = \{ s \in S \mid s \notin f(s) \}$ is not in Image(f)

negation

almost "self-referencing"

$s \quad f(s)$

about containment.

Russell's Paradox: If $R = \{ S \text{ set} \mid S \notin S \}$

you get a contradiction with either $R \in R$ or $R \notin R$

Resolution (you have ask a set theorist or logician)

"the set of all sets" is too big to be set

so $\{ S \text{ sets} \mid \text{blah} \}$ ~~is~~ can be too large to be set

$\{ S \mid S \notin S \}$ ← "self-referency" + "negation"

§6 Handout gives a few more:

Paradox (3): What is "the smallest positive integer not described by a phrase in English of at most 100 words" ?

one hundred

(less than 100 words)

Say it's the number

$$n = 157392416 3 \ldots 279.$$

"Blandy's Paradox" (probably due to Russell)

self-reference + "not" = negation.

Paradox (4), §6:

Leslie write about (and only about) those people who do not write about themselves.

Does Leslie write about themself?

If yes – contradiction
if no – "

Also "barber paradox"

Later: "Halting problem" is undecidable

Proof: Assume it is decidable, and get a contradiction via "self-referencing" + "negation"

Chapter 1 in textbook by Sipser:

Regular Languages $\longleftrightarrow$ languages described by $\underbrace{\text{"regular expressions"}}_{\text{Section 1.3}}$

Idea: $\Sigma = \{a, b\}$,

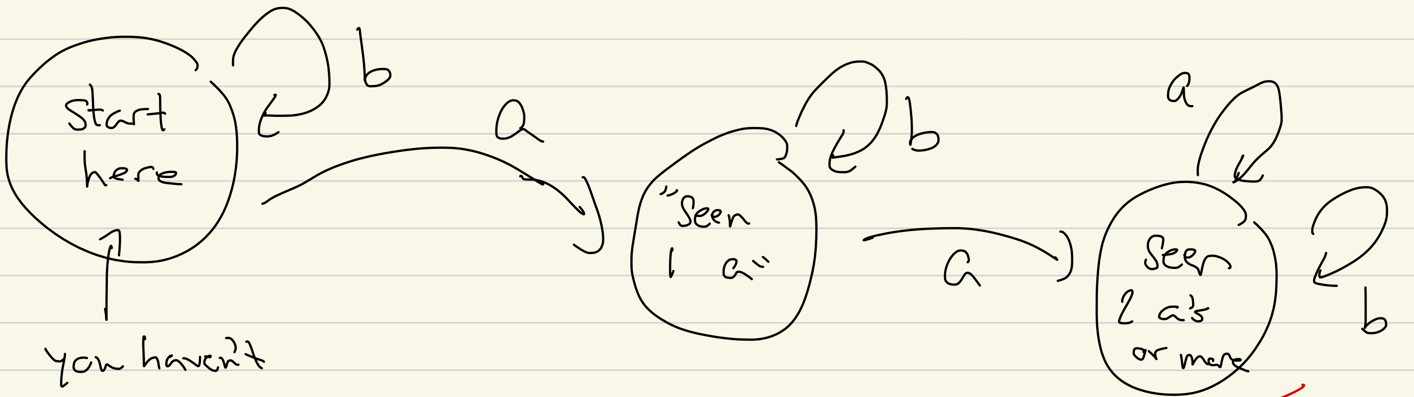
Let $L = \{ w \in \Sigma^* \mid w \text{ has at least 2 } a\text{'s (in its set of symbols)} \}$

$= \{ aa,\ aab,\ baa,\ aba,\ bbabbbab, \ldots \}$

Our algorithm (informally):

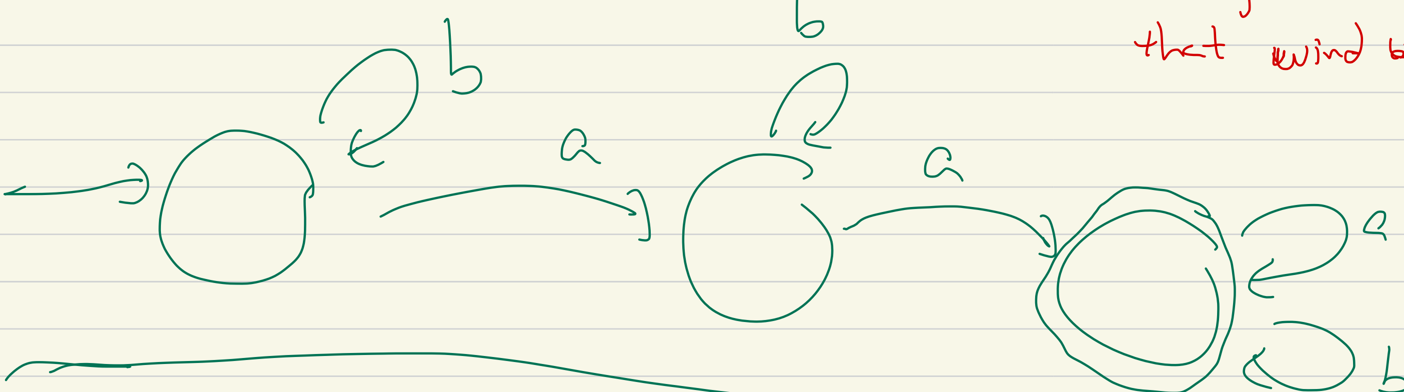
- read each symbol (letter) of w, one by one

$$w = \sigma_1 \sigma_2 \cdots \sigma_k \qquad \sigma_i \in \Sigma = \{a, b\}$$

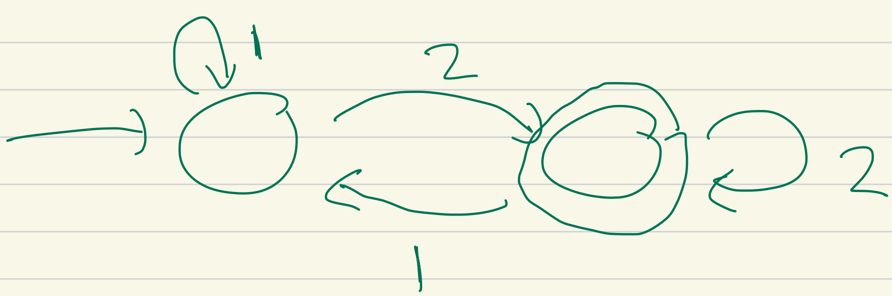


you haven't seen any a's

accept string/words that wind up in



Say $L = \{ w \in \{1,2\}^* \mid \text{end in } 2 \}$ $\quad \Sigma = \{1, 2\}$



Formally: A finite automaton is 5-tuple

$$(Q, \Sigma, \delta, q_0, F):$$

a set "set of states"

alphabet of language

$q_0 \in Q$ "initial state"

"final states" "accepting states"

$$\delta : Q \times \Sigma \to Q$$

idea: $(q, \sigma) \mapsto$ what is the new state that you move to when in state q you see a σ

Each finite automaton (DFA) "deterministic", $M = (Q, \Sigma, \delta, q_0, F)$

"recognizes" $L_M = \{ \underline{w} \in \underline{\Sigma}^* \mid$ following the DFA, you finish at the last symbol of w in a state in F $\}$

Def: Σ alphabet, $L \subset \Sigma^*$, L is regular iff L is recognized by some finite automaton.

Otherwise we say L is non-regular

Rem: DFA $M = (Q, \Sigma, \delta, q_0, F)$

Q: finite

Example: Later $L = \{ a^m b^n \mid m = n \}$ is non-regular

$a^m = \underbrace{a \dots a}_{m}$, $b^n = \underbrace{b \dots b}_{n}$

Can specify a DFA by

(1) $(Q, \Sigma, \delta, q_0, F)$ written out

(2) By graphs drawn using notation in [Sip]

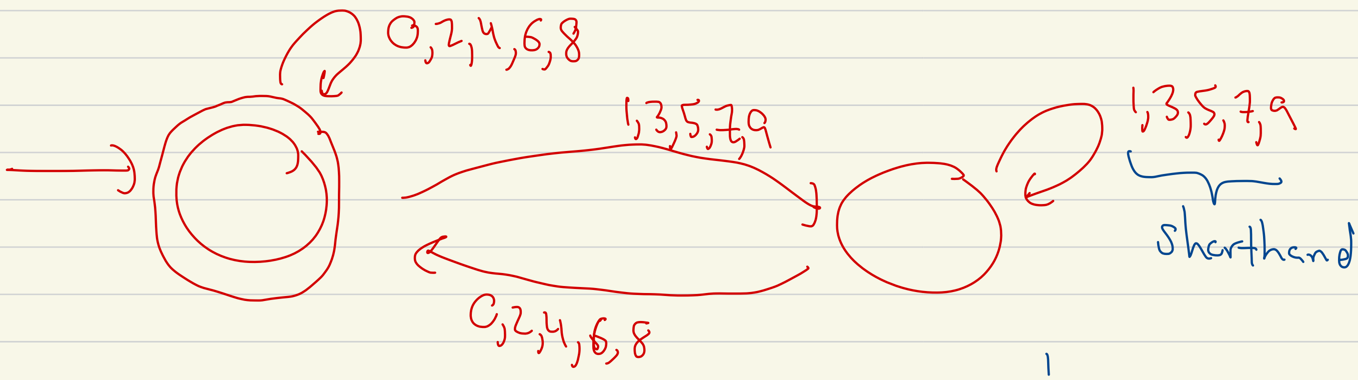
Convention: $\text{EVEN} = \{ w \in \{0, \dots, 9\}^* \mid w \text{ represents an even number} \}$

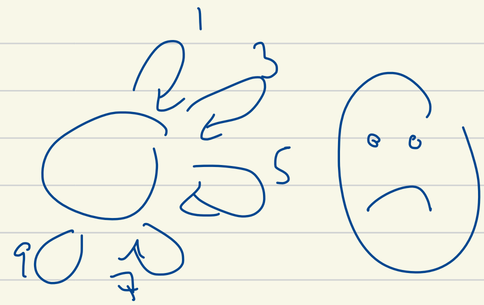
$= \{ 0, 2, 4, 6, \ 02, \ \varepsilon, \dots \}$

02 ? ε ?

you have to specify

Simple DFA for EVEN = { ε, 0, 2, 4, 6, 8, 00, 02, 04, ..., 10, 12, ... }





Breakout rooms:

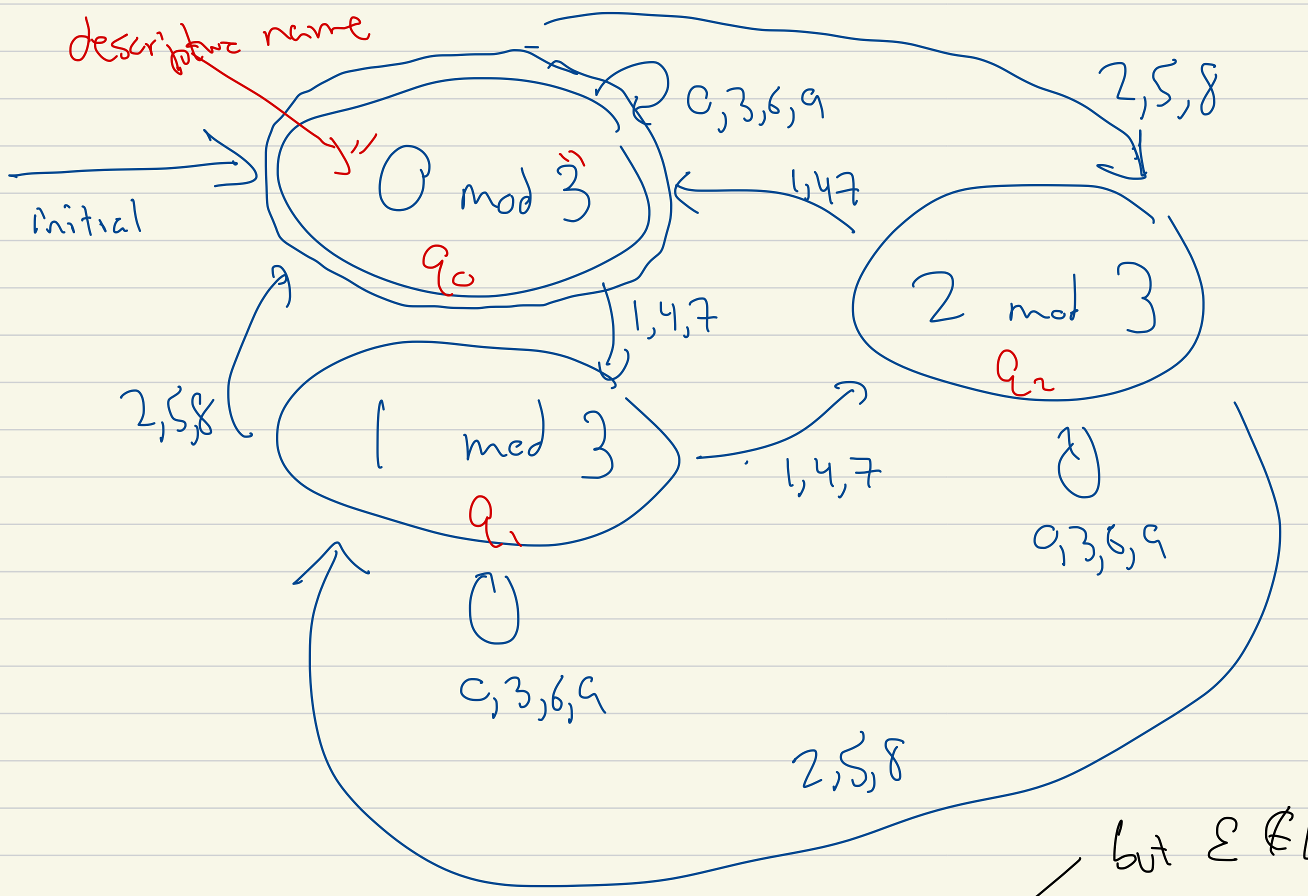
Problem 4, then problem 3 of today breakout room problem

Roughly 8-10 minutes

Problem 4:

$$L_{\text{DIV BY 3 EASY}} = \{\varepsilon, 0, 3, 6, 9, 00, 03, 06, 09, \dots\}$$

Build DFA recognizing $L_{\text{DIV BY 3 EASY}}$



but $\varepsilon \notin L$

Q: How to modify for $L = \{0, 3, 6, 9, 03, 06, \dots\}$

New: Survey under "Quiz"

for the canvas webpage

"Survey After Class on Sept 24"