

CPSC 421/501

Sept 15

- Review strings/words & languages
 - State thm: there is no surjection $\Sigma^* \rightarrow \text{Power}(\Lambda^*)$ for alphabets Σ, Λ and explain relevance
 - Prove this thm using set theory
 - Russell's Paradox & Other paradoxes/thms with negation + self-references
- today and/or Thursday
- on Thursday

All this is in the first handout!

Self-referencing, Uncountability, and Uncomputability

We'll go around 10:20 (50 minutes), breakout rooms in Zoom

for some problems (typically from 2019 homework

or 2019 webpages - review midterm
- review final

meet back; discussion/question

I'll post office hours for tomorrow by this afternoon.

Homework teams of up to 4 students;

some problems written up by each student

" " " " by the team

Last time } start of handout
 } Ch. 0 of textbook: Introduction to the
 Theory of Computing, By M. Sipser
 3rd Edition

An alphabet, Σ , is any finite, non-empty set.

The elements of Σ are called symbols or letters
 ↑ ↑
 textbooks very common in
 the literature

$\Sigma^k = k$ -tuples of elements of Σ

$\Sigma = \{a\}, \Sigma^4 = \{(a,a,a,a)\}$ more simply $\{aaaa\}$
 or $\{a^4\}$

$\underbrace{\Sigma \times \Sigma \times \dots \times \Sigma}_{k^{\text{th}} \text{ Cartesian product}}$

ordered
 tuples

$\Sigma^0 = \{\epsilon\}$ $\epsilon =$ empty string

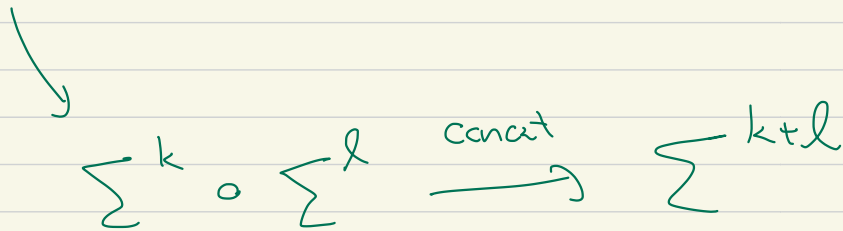
$\Sigma = \{a,b\}, \Sigma^2 = \{aa, ab, ba, bb\}$

↑
 literally ab is really (a,b)

{ string:

Concatenation: $aba \circ bb$

$= ababb$



$\xi \circ aba$

$= aba$

Σ is an alphabet

"set of strings over Σ " notation Σ^* is just

$$\Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots = \bigcup_{m=0}^{\infty} \Sigma^m$$

Claim: Any C-program is a string over

ASCII alphabet,

$|\text{ASCII}| = 256$, has $a, \dots, z, A, \dots, Z, 0, 1, 2, \dots, 9$

This is mostly true, (it does depend on how you interpret a C program — what built in libraries, etc.)

Similarly, any book in English is an ASCII string (ignore illustrations, etc.)

There are many notions of an algorithm:

① C program ② Javascript program

③ Turing machine ④ Deterministic Finite Automaton
(Ch. 3) DFA (Ch. 1)

All be written — with some understanding — as finite strings over some fixed finite alphabet.

e.g. indentation — you have to agree on some convention

Turing machines & DFAs as well

Compound words : Antidestablishmentarionistm

Algorithms

Interpretation
↔

finite length
strings over some
alphabet Σ

Language over an alphabet, Σ , is a subset of Σ^* .

e.g.

$\Sigma = \{0, 1, \dots, 9\}$, languages over Σ include:

PRIMES = $\{2, 3, 5, 7, 11, 13, 17, 19, \dots\} \subset \Sigma^*$
↳ (literally 17 mean (1,7))

SQUARES = $\{1, 4, 9, 16, 25, 36, \dots\} \subset \Sigma^*$
 $= \{0, 1, \dots, 9\}^*$

Over $\Sigma = \{a, b\}$

"PALINDROMES OVER $\{a, b\}$ "

= $\{\epsilon, a, b, aa, bb, aaaa, \dots\}$

not ab or ba or acb

$$= \left\{ \sigma \in \Sigma^* \mid \sigma^{\text{reverse}} = \sigma \right\}$$

where

$$\sigma = (\sigma_1, \dots, \sigma_k), \quad \sigma^{\text{reverse}} = (\sigma_k, \sigma_{k-1}, \dots, \sigma_1)$$

or $\sigma_1 \dots \sigma_k \quad \quad \quad = \quad \sigma_k \sigma_{k-1} \dots \sigma_1$

=

A language: e.g. PRIMES
SQUARES
PALINDROME OVER Σ

can be viewed as decision problem

formally: A decision problem over Σ (where Σ is an alphabet)

is any subset of Σ^* . The set of all languages

$$\underline{\text{over } \Sigma} = \text{Power}(\Sigma^*)$$

where S any set, $\text{Power}(S) =$ 'set of all subsets of S '

Theorem: For any alphabets Σ, Λ ,

any map $f: \Sigma^* \rightarrow \text{Power}(\Lambda^*)$

is not surjective, i.e. there is some $L \in \text{Power}(\Sigma^*)$ s.t. there is no $\sigma \in \Sigma^*$ with $f(\sigma) = L$.

Application: For the notions of "algorithm" in CPSC 421/501 (DFA's, Turing machines, etc.) there are decision problems that can't be solved:

$f: \Sigma^* \rightarrow \text{Power}(\Sigma^*)$
 interpret as an algorithm to solve decision problem
 { invalid descriptions or decides some language over Σ }

Goal is to prove this, and ~~this~~ will it will be useful to speak countable / uncountable sets.

Set: $\{1, 2, 3\}$, $\{a, b\}$, $\{\text{languages over } \Sigma\}$
 $= \text{Power}(\Sigma^*)$

$f: S \rightarrow T$ map of sets S, T is

(1) injective if $s_1 \neq s_2, s_1, s_2 \in S$ then

$f(s_1) \neq f(s_2)$; e.g. student ID numbers —

$\{\text{UBC students}\} \rightarrow \{0, \dots, 9\}^8$ { "injective" "into" same

(2) surjective if $\forall t \in T \quad \exists s \in S$

for all t in T there is some $s \in S$

st. $f(s) = t$.

(3) bijective if it is injective & surjective

(also called one-to-one correspondence)

e.g. $\{1, 2, 3\} \xrightarrow{f} \{ \text{😊}, \text{😞}, \text{😏} \}$

$f(1) = \text{😏}, f(2) = \text{😊}, f(3) = \text{😞}$

e.g. surjective

Joel gardens:

Monday \rightarrow Mint

Tuesday \rightarrow Basil

Wed \nearrow

Th \nearrow

Fr \nearrow

Sat, Sun \rightarrow relax

hit everything
 \leftarrow in T

$$f: \{ \text{Monday}, \dots, \text{Sunday} \} \rightarrow \{ \text{Mint}, \text{Basil}, \text{relax} \}$$

$$f: S \longrightarrow T$$

↓ ↓
"domain" "range"

f could map

$$f: \{ \text{Mon}, \dots, \text{Sun} \} \rightarrow \{ \text{Mint}, \text{Basil}, \text{relax}, \text{lettuce} \}$$

if I never
work or lettuce,
bad...

Say that S and T have the same size

if there is a bijection $f: S \rightarrow T$.

We say that T is larger than S if there is no
surjection $S \rightarrow T$.

S, T finite, and $|S| = \#$ of elements
in S

\exists bijection $S \rightarrow T$ iff $|S| = |T|$

but

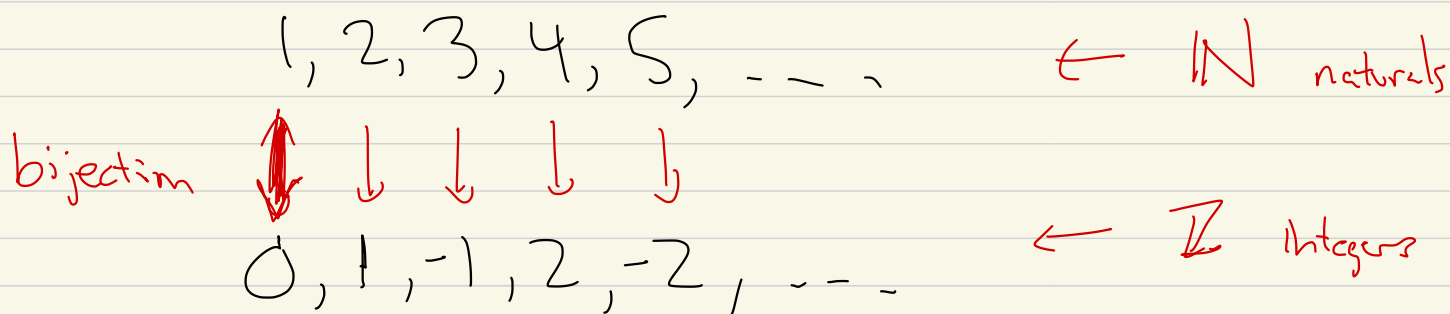
$$\mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$$

$$\mathbb{Q} = \mathbb{Q}_{>0} = \{\text{positive rationals}\}$$

$$\mathbb{Z} = \{0, 1, -1, 2, -2, \dots\}$$

} all have
same
size

e.g. even though $\mathbb{N} \subset \mathbb{Z}$
proper



Questions for discussion (random breakout)

① HW:

- do the followings sets have the same size as \mathbb{N}

Hint: Has the same size - $\{\text{functions from } \{0, 1\} \rightarrow \mathbb{N}\}$

this one ~~doesn't~~ - { functions from $\mathbb{N} \rightarrow \{0,1\}$ }

② Give an "intuitive example" of injection / surjection

③ Come up with a surprising injection / surjection / bijection

④ If f, g are surjections, $f: S \rightarrow T$
 $g: T \rightarrow U$

is $g \circ f: S \rightarrow U$ and why? can you prove?

—
You can currently do ②, ③, ④ ←

Next time we will do ①;