GROUP HOMEWORK 8, CPSC 421/501, FALL 2020

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

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(1) Show that any Boolean function $f = f(x_1, \ldots, x_n)$ on Boolean variables $x_1, \ldots, x_n$ can be written as:
  (a) a DNF formula $c_1 \vee \ldots \vee c_s$ where $s \leq 2^n$ and each c_i is the AND of n literals;
  (b) a CNF formula $c_1 \wedge \ldots \wedge c_s$ where $s \leq 2^n$ and each c_i is the OR of n literals.

  [Hint: once you do the first part, you can do the second part by considering a DNF for $\neg f$.]

(2) Show that 3COLOUR is NP-complete, using the hints in the textbook for Problem 7.29.

(3) Assume that 3COLOUR is NP-complete. Show that 4COLOUR is NP-complete (where 4COLOUR is the set of descriptions of graphs that are colourable with 4 colours).

(4) Let 4SAT be the descriptions of Boolean formulas in 4CNF that are satisfiable (a 4CNF is the AND of clauses, each of which is the OR of 4 literals). Show that 4SAT is NP-complete.

(5) If L_1 can be reduced to L_2 in time $O(n^3)$, and L_2 can be reduced to L_3 in time $O(n^5)$, what can you say about the time that it takes to reduce L_1 to L_3? Explain. [Hint: the answer is $O(n^{15})$, not generally smaller.]

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