(1) (a) Let $L$ be a regular language over $\Sigma$, and let $w \in \Sigma$. Show that $L \cup \{w\}$ is regular.
(b) Let $L$ be a regular language over $\Sigma$, and let $w \in \Sigma$. Show that $L \setminus \{w\}$ is regular.
(c) Let $L$ be a nonregular language over $\Sigma$, and let $w \in \Sigma$. Show that $L \cup \{w\}$ is nonregular.
(d) Let $L$ be a nonregular language over $\Sigma$, and let $w \in \Sigma$. Show that $L \setminus \{w\}$ is nonregular.

(2) Prove the following statements, or prove that they are false by giving a counterexample.
(a) The union of two regular languages is regular.
(b) The union of two nonregular languages is regular.
(c) The union of a nonregular language and a regular language is nonregular.
(d) The concatenation of a nonregular language and a regular language is nonregular.

(3) Give a formal description (by a state diagram, or table of $\delta$ values, or a list of $\delta$ values) of a Turing machine that recognizes the language over $\Sigma = \{0, 1\}$ given by

$$L = \{w \in \{0, 1\}^* \mid w \text{ has more 0's than 1's}\}.$$
Make sure to explain how your machine works, and what each state “means” in terms of your algorithm; make sure that it is clear what are the values of $Q, \Gamma, q_0, q_{\text{accept}}, q_{\text{reject}}$.