GROUP HOMEWORK 4, CPSC 421/501, FALL 2020

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

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(1) Let L be the language of strings that contain ab as a substring. Use the Myhill-Nerode theorem to show that any DFA accepting L has at least 3 states.

(2) (a) Let L be the language described by the regular expression $(0\cup 1)^*1(0\cup 1)^3$, i.e., the words of length at least 4 whose 4th last symbol is 1. Write an NFA with 5 states that recognizes L.
(b) Generalize part (a) to the language $(0\cup 1)^*1(0\cup 1)^k$ for any $k\in\mathbb{N}$.

(3) (a) Let L be the language described by the regular expression $(0\cup 1)^*1(0\cup 1)^1$, i.e., the words of length at least 2 whose 2nd last symbol is 1. Use the Myhill-Nerode to find minimum number of states needed in a DFA that recognizes L. [Hint: Show that $\mathrm{AccFut}_L(u)$ for $u = 00, 01, 10, 11$ are all different. Are there any other possible values for $\mathrm{AccFut}_L(u)$ with u of length 3 or greater? What about for u of length 0 or 1?]
(b) Give a DFA that recognizes L with fewest possible states, and explain how your DFA works.
(c) Generalize parts (a,b) to the language $(0\cup 1)^*1(0\cup 1)^k$ for any $k\in\mathbb{N}$.

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The following is a bonus question, worth an additional 10 points out of 100.

(4) Let L be a regular language over an alphabet Σ, and let L' be any language (not necessarily regular) over Σ. It the language

$$L'' = \{w \in \Sigma^* \mid \text{for some } w' \in L' \text{ we have } ww' \in L\}$$

necessarily regular? [Hint: One way to do this is to write $\mathrm{AccFut}_{L''}(u)$ in terms of certain values of AccFut_L.]

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