GROUP HOMEWORK 2, CPSC 421/501, FALL 2020

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

In these exercises, "the handout" refers to the article "Self-referencing, Uncountability, and Uncomputability" on the 421/501 homepage.

(1) Say that S, T are finite sets. What can you say about how $|S|$ and $|T|$ compare when:
 (a) there exists an injection $S \to T$?
 (b) there does not exist an injection $S \to T$?
 Briefly explain why (you do not need to give a formal proof).

(2) Let $f\colon [4] \to \mathrm{POWER}([4])$ be given by $f(1) = \emptyset$, $f(2) = [4]$, $f(3) = \{1, 4\}$, $f(4) = \{2, 3, 4\}$. What is the set $T = \{s \in [4] \mid s \notin f(s)\}$?

(3) Let $f\colon [4] \to \mathrm{POWER}([4])$ be any map such that $f(1) = \emptyset$, $f(2) = \{1, 2, 3\}$, and let $T = \{s \in [4] \mid s \notin f(s)\}$. Explain why $f(1)$ cannot equal T (regardless of the values of $f(3)$ and $f(4)$), just using the fact that $1 \notin f(1)$. Explain why $f(2)$ cannot equal T, just using the fact that $2 \in f(2)$.

(4) Assume Problem 1 of the Individual Homework 2 is true. Show that $\mathrm{POWER}(\{a, b\}^*) \cup \mathbb{N}$ is uncountable.

Research supported in part by an NSERC grant.

(5) (Recall that if S, T are sets, T^S refers to the set of functions from S to T.) Justify your answers to the following.
  (a) Describe an injection $[2]^{\mathbb{N}} \to [3]^{\mathbb{N}}$.
  (b) Describe a surjection $[3]^{\mathbb{N}} \to [2]^{\mathbb{N}}$.
  (c) Describe a bijection $[2]^{\mathbb{N}} \to [4]^{\mathbb{N}}$.
  (d) Assume the following is true: "if there exists is a surjection $S \to T$, and there exists a surjection $T \to S$, then there is a bijection $S \to T$." Using this, show that there exists a bijection $[2]^{\mathbb{N}} \to [3]^{\mathbb{N}}$.

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