GROUP HOMEWORK 1, CPSC 421/501, FALL 2020

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Please note:
(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) Homework that is difficult to read may not be graded.
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

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In these exercises, "the handout" refers to the article "Self-referencing, Uncountability, and Uncomputability" on the 421/501 homepage.

(1) Which of the following maps are injections (i.e., one-to-one), which are surjections (i.e., onto), and which are both (i.e., one-to-one correspondences)? Justify your answer.
   (a) $f \colon \mathbb{N} \to \mathbb{N}$ given by $f(x) = 4x + 2$.
   (b) $f \colon \mathbb{N} \to \mathbb{N}$ given by $f(x) = x^2$.
   (c) $f \colon \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x^4$.

(2) If $f \colon S \to T$ is surjective, and $g \colon T \to U$ is injective, is $g \circ f$ necessarily surjective? Justify your answer.

(3) Give a map $f \colon \{0, 1\}^*$ to $\mathbb{N}$ that is a bijection, and justify your answer (i.e., explain why your choice of f is a bijection). [There are many ways solve this problem; some ways are simpler than others.]

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